The Performance of Policy Rules in Heterogeneous-Agent Models with Aggregate Shocks

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Ce qu'ils font dans ce papier?

- By paper, I mean a composite of their JEDC (2018) paper (henceforth, BKM) and new applications presented today.
- BKM (2018) proposed a novel solution algorithm for solving a Krusell-Smith (1998) type of models.
- Objective is to improve on a seminal Krusell and Smith (1998) algorithm (henceforth, KS).
- *Idea of KS's algorithm*: represent an infinite dimensional object – wealth distribution – with a finite set of moments, usually, a mean is sufficient.

$$\Downarrow$$

"Approximate aggregation".

Ce qu'ils font dans ce papier?

- *Idea of BKM's algorithm*: avoid iterating on distributional moments by assuming linearity of aggregate choices in aggregate shocks.

$$\Downarrow$$

"Approximate linearity".

- *Alternatives in the literature:* Reiter (2009), Ahn, Kaplan, Moll, Winberry, Wolf (2017) use model reduction techniques which are complicated and may run out of memory.
- BKM's algorithm is simple, intuitive, requires minimal additional programming efforts relative to finding a steady state (Aiyagari, 1994).

It has all ingredients to become as popular as KS (1998) algorithm (or more)!

Policy Analysis with BKM's Algorithm

- One important advantage of the new algorithm:
  - It may work for a larger set of applications.
    – BKM solve a HA model with two continuous aggregate shocks and valued leisure.
  - Importantly, it suits for analyzing the effects of different policies on distributions.
- Recent interest in studying the effects of different policies on distributions:
  – HANK of Kaplan, Moll and Violante (2018).
- Whole distribution matters (may need to keep track of higher dimensional representation of the distribution).
  $\Longrightarrow$ KS's algorithm will not work.

BKM's Algorithm: Representative Agent Model

Solution procedure:

- Start in the steady state.
- At $t = 0$, agg. productivity $z_0 \uparrow$ due to a one-time, unexpected aggregate productivity shock $\epsilon_{z,0}$ ("MIT" shock, $\epsilon_{z,0} \to 0$). No more shocks in the future.
- Choose time horizon T for finding a solution (the one that ensures a return to the steady state).
- Solve for a transition path of variables $x = (y_t, c_t, k_{t+1}, h_t, b_{t+1})$ using a path solving method (e.g., extended path method of Fair and Taylor, 1983).
- Treat the resulting path for x as an impulse response – a linear numerical derivative, given by a period-specific *vector* of constants $\{\gamma_0, ..., \gamma_T\}$

$$x_0 = \gamma_0 \epsilon_{z,0}, \; x_1 = \gamma_1 \epsilon_{z,0}, \; x_2 = \gamma_2 \epsilon_{z,0}, \ldots$$

- IRF resulted from a fully nonlinear model assuming certainty equivalence.⇒IRF captures some model's nonlinearity.

BKM's Algorithm: Representative Agent Model

Idea of simulations:

- If the only shock is $\epsilon_{z,0}$, $x_1 = \gamma_1 \epsilon_{z,0}$.
- If the only shock is $\epsilon_{z,1}$, $x_1 = \gamma_0 \epsilon_{z,1}$.
- With two shocks $\epsilon_{z,0}$ and $\epsilon_{z,1}$, write x_1 as a superposition of IRFs:

$$x_1 = \gamma_1 \epsilon_{z,0} + \gamma_0 \epsilon_{z,1}$$

- Why? By assumption: decisions are linear in aggregate states.

BKM's Algorithm: Representative Agent Model

Simulation procedure:

- Draw a sequence of productivity shocks $\{\epsilon_{z,t}\}_{t=0}^{T}$ and simulate the model by using the IRF:

$$x_t = \gamma_0 \epsilon_{z,t} + \gamma_1 \epsilon_{z,t-1} + \gamma_2 \epsilon_{z,t-2} + ... + \gamma_t \epsilon_{z,0} \qquad (*)$$

- With *two exogenous variables* z and χ, MIT shocks are $\epsilon_{z,0}$ and $\epsilon_{\chi,0}$.
  – The path-solving method delivers $\{\gamma_0, ..., \gamma_T\}$ and $\{\gamma_0^{\chi}, ..., \gamma_T^{\chi}\}$.
  – To simulate the model, sum up the IRF:

$$x_t = \gamma_0 \epsilon_{z,t} + \gamma_0^{\chi} \epsilon_{\chi,t} + ... + \gamma_t \epsilon_{z,0} + \gamma_t^{\chi} \epsilon_{\chi,0} \qquad (**)$$

Comment 1: It Is Not Linearization

- BKM refer to their solution approach as linearization.
- In what sense is it linearization?
  – Not in a standard sense.
  – Equations (*) and (**) are their **assumptions**.
  – Standard linearization is a process of finding Taylor's expansions.
- To verify whether the linearity assumptions are satisfied, they proposed some tests of linearity.
- However, these tests can just indicate whether linearity is *roughly* satisfied in their solution.

Comment 2: Linearity Rules Out Interesting Cases

- To check the linearity (*) and (**), two tests proposed:
  - *scalability* (scale the initial shock up, and see whether a new IRF shifts in a parallel way);
  - *additivity* (two shocks occur at the same time versus they occur separately).
- The linearity assumption may rule out many interesting cases:
  – γ´s depend on the size of $\epsilon_{z,t}$;
  – interaction between different shocks affects the behavior;
  – many shocks;
  – uncertainty shocks as in, e.g., Bloom, 2009;
  – volatility changing over time.

Comment 3: Solution & Simulation Procedures

General rule in numerical analysis:

- *"Do in the simulation procedure what you did in the solution procedure"*.
- When solving the model, BKM assume a one-time unexpected (MIT) shock.
- But when simulating the model, they draw a standard sequence of shocks $\{\epsilon_{z,t}\}_{t=0}^{T}$ such that,

(i) shocks happen in every period of time;

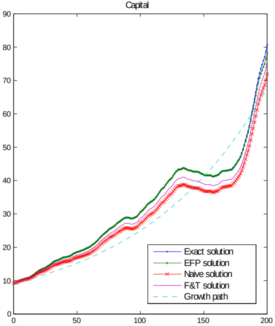
(ii) shocks are not small (0.01 of std dev.) but realistically calibrated (1 std dev.).

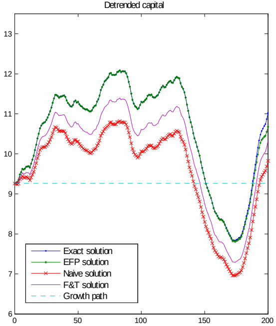
Comment 3 (i): Agents are Naive

- Agents are naive:
  – "unaware" of future productivity changes;
  – have expectations that are systematically differ from those of "aware" agents;
  – confronted with changes later.
- In the nonlinear model, decisions of naive agents and fully-rational agents may differ substantially; see Maliar, Maliar, Tsener and Taylor (2015) (henceforth, MMTT).
- MMTT (2015) proposed a solution algorithm for solving nonstationary models.
- One application – optimal growth model with labor augmenting technological progress and exogenous labor productivity shocks.
- *"Naive solution"* neglects a connection between the decision functions of different periods.

Comment 3 (i): Agents are Naive

Difference between the exact and naive solutions is 10%.





Conclusion of MMTT (2017): approximating expectation functions accurately is critical for constructing accurate solutions.

Comment 3 (ii): Too Large Increase in the Size of Shocks

- When solving the model, BKM assume MIT shocks.
- An MIT shock must be infinitesimal. Why is it important? *"as one should view it as of the size required to produce a numerical derivative: the entire impulse response ... should thus be viewed as a numerically computed derivative of the initial shock".*
- But they simulate the model not with small shocks $(0.01 \cdot \sigma_z)$ but standard RBC shocks $(1 \cdot \sigma_z = 0.007)$.
- This is justified by the linearity assumption (*):
  – Scale the shock up by 100, output will increase by a factor of 100!
- What about a factor of 10^6?
- Is it reasonable to expect that aggregate choices will not be affected non-trivially under such large increases?

Comment 4: Unknown Convergence Properties

1. BKM approximate the solution to the infinite-horizon model with a solution to a T-period model ($T = 350$).
   Implicitly, they rely on **turnpike theorem**:
   "*The trajectory of the finite horizon economy asymptotically converges to that of the infinite horizon-economy as the horizon increases*".

- It holds for the optimal growth model, see MMTT (2017), but no results are known for a model with labor, bonds, externalities.

2. Path solving methods do not check **Blanchard-Kahn conditions** (like Dynare reports for perturbation).
   - We will not know anything about forward stability properties.
   - If the algorithm breaks down, it will be unclear whether it happens because of some bug in the code or because no forward stability.

Heterogeneous Agents Model

- Consider the standard KS (1998) HA model.
- Exogenous aggregate productivity level (z_t) and idiosyncratic labor productivity level (ε_t); Cobb-Douglas technology.
- At $t = 0$, aggregate productivity $z_0 \uparrow$ due to a one-time, unexpected aggregate productivity shock $\epsilon_{z,0}$ ("MIT" shock, $\epsilon_{z,0} \longrightarrow 0$).
- No more shocks in the future.
- BKM consider $T = 350$ periods. Assume that this T is long enough for the economy to return to the steady state.
- *Comment:* Again, BKM use implicitly here turnpike theorem.
    - No turnpike results are obtained for this incomplete-market model.
    - For example, Maliar and Taylor (2018) showed that turnpike theorem does not generally hold for the standard 3-equation new Keynesian model.

BKM's Algorithm: Heterogeneous-Agent Model

Solution Procedure:

- Assume the capital-labor ratio for every period of time, $\left\{\left(\frac{K}{L}\right)_t\right\}_{t=0}^{T}$. $\Longrightarrow$ get $\{r_t, w_t\}_{t=0}^{T}$.
- Use $\{r_t, w_t\}_{t=0}^{T}$ to solve for transition path from an individual problem backward:
  – Starting from the steady-state value function $V^T = V^{ss}$, compute time-dependent functions $\{V_t(k_t, \varepsilon_t)\}$
  – ε_t = idiosyncratic labor-productivity shock.
  – Here $V_t(\cdot)$ is time-dependent because r_t, w_t are time-dependent.
  – *Note:* wealth distribution is not explicitly included as an argument of V_t
- Starting from the steady state distribution, simulate the distributions forward (with histograms) using individual policy rules and Markov transition matrix of probabilities.

BKM's Algorithm: Heterogeneous-Agent Model

Solution Procedure:

- Compute new $\left\{\left(\frac{K}{L}\right)_t\right\}_{t=0}^{T}$. Update the guess on $\left\{\left(\frac{K}{L}\right)_t\right\}_{t=0}^{T}$ by using homotopy.
  – Iteration on the ratio's path is similar in spirit to iteration on steady-state K in Aiyagari (1994) model.
- Treat the resulting path for a generic aggregate statistics x as an impulse response. Get $\{\gamma_0, ..., \gamma_T\}$.

BKM's Algorithm: Heterogeneous-Agent Model

Simulation Procedure:

- Assume the economy experiences recurring aggregate shocks to productivity z. Draw shocks $\{\epsilon_{z,t}\}_{t=0}^{T}$ and simulate the model by using the IRF:

$$x_t = \gamma_0 \epsilon_{z,t} + \gamma_1 \epsilon_{z,t-1} + \gamma_2 \epsilon_{z,t-2} + \ldots + \gamma_t \epsilon_{z,0}.$$

- *Advantage:* can easily find the evolution for any distributional statistics; e.g., $x =$ 3rd percentile of the wealth distribution.

Current Presentation of the Algorithm

- Did the new algorithm achieve its goal of improving on KS algorithm?
  – We do not know for sure: no direct comparisons with KS (1998) algorithm.
  – In general, more comparisons with other algorithms (Reiter, 2009, Ahn et al., 2017) would be very helpful.
- Verification that the method works robustly and is accurate.
  – Closeness of second moments (volatilities, correlations, autocorrelations) is not a valid verification of the accuracy; see the special JEDC (2010) issue.
- Better explanation to the users what the algorithm does.
  – Many computational details are omitted.
  – BKM say that in the agent's problem "time" is a state variable. But they just mean time-dependent functions.

New Applications: RA version

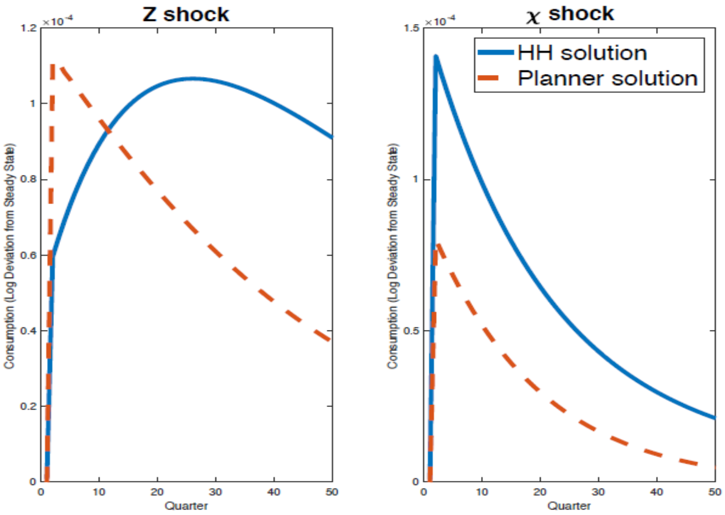
- Two shocks (to preferences and productivity).
- Aggregate consumption affects both factor utilization and employment.
- The planner solves:

$$\max_{c_t, k_{t+1}, h_t} \sum_{t=0}^{\infty} \beta^t \left(\chi_t \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{1}{1+\frac{1}{\theta}} e_t h_t^{1+\frac{1}{\theta}} \right)$$

$$\begin{aligned} c_t + k_{t+1} &= (1-\delta) k_t + z_t \phi(c_t) k_t^{\alpha} (e_t h_t)^{1-\alpha} \\ e_t &= \zeta(c_t, e_{t-1}). \end{aligned}$$

- $z_t \uparrow \implies$ the planner wants higher consumption (not full consumption smoothing under $\phi'(c_t) \neq 0$) $\implies$ "de-stabilizing".
- $\chi_t \uparrow \implies c_t \uparrow$ less because it does not decrease enough marginal utilization $\phi'(c_t) \implies$ "stabilizing".

New Applications: RA version



New Applications: HA version & cyclical transfers

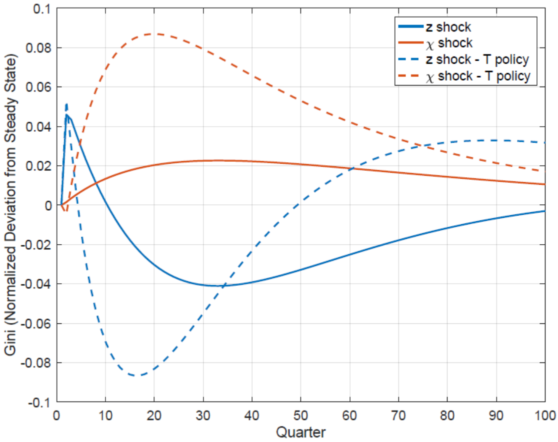
- Agent's budget constraint now includes labor tax $(1-\tau)\,\omega_t w_t h_t$ and transfer T_t.
- Cyclical, deficit-financed lump-sum transfers:

$$
\begin{aligned}
T_t &= \widehat{C}_t + 0.005\widehat{B}_t + \tau w_t \int e_t h_t d\omega_t \\
B_{t+1} &= (1 + r_t - \delta)\, B_t - T_t + \tau w_t \int e_t h_t d\omega_t
\end{aligned}
$$

- IRFs for two agg. shocks, and for two versions of the model look very different.

New Applications: HA version & cyclical transfers

The evolution of Gini coefficient over time:



Different policies have dramatically different effects on wealth inequality!

HA version: Further Work

1. BKM investigate the effect of fiscal policy on aggregate measures of inequality (Gini).
   – Who gains and who loses from such policy? Disaggregate across poor and rich, skilled and unskilled.
   – Calculate welfare.
2. BKM now compare the solutions to the RA and HA versions using IRFs.
   – Interesting to separate RA effects from HA effects, in spirit of HANK by Kaplan et al. (2017).
3. Introduce more agents' heterogeneity.

Conclusion

- BKM's
  algorithm is a highly desirable alternative for policy analysis.

- Comparisons with other alternatives (Reiter, 2009, Ahn et al. 2017) should convince the users.

- More work is needed to understand

  – when it is useful (inequality constraint; stability of solutions is a fact; few agg. shocks; the effect of uncertainty is small)

  – and when it is not useful (...).

Thank you!