Household Savings and Monetary Policy under Individual and Aggregate Stochastic Volatility

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Motivation

- Four major developments in macroeconomics:
  - heterogeneity in income or productivity and assets with differing liquidity (machine, shares, liquid bonds)
  - aggregate (and idiosyncratic) uncertainty done right
  - global solutions
  - redistribution
- These are usually done in isolation.

**This paper**: We do all four in one framework
HANK Model

- **Households**
  - Three types of assets: bonds (liquid), shares and machines (illiquid)
  - Three borrowing constraints, one for each asset
  - Idiosyncratic shocks to productivity level and volatility
  - Heterogenous labor

- **Firms**
  - CRS with machines and labor
  - Aggregate shocks to TFP level and volatility
  - Sticky prices (Rotemberg)

- **Government**
  - Fiscal policy (lump-sum taxes, for now)
  - Monetary policy (Taylor rule with ZLB)
Levels and Uncertainty Shocks

- Household productivity
  
  \[ \eta_{\ell,t} (j) = \rho^\ell \eta_{\ell,t-1} (j) + \exp (\sigma_{\ell,t-1}) \varepsilon_{\ell,t} (j) \]
  
  uncertainty:
  
  \[ \sigma_{\ell,t} = \rho^\sigma_\ell \sigma_{\ell,t-1} + \sigma_{\ell} \varepsilon_{\sigma_\ell,t} \]

- Aggregate TFP
  
  \[ \eta_{\theta,t} = \rho^\theta \eta_{\theta,t-1} + \exp (\sigma_{\theta,t-1}) \varepsilon_{\theta,t} \]
  
  uncertainty:
  
  \[ \sigma_{\theta,t} = \rho^\sigma_{\theta} \sigma_{\theta,t-1} + \sigma_{\sigma_{\theta}} \varepsilon_{\sigma_{\theta},t} \]

where \( \varepsilon_{\ell,t}, \varepsilon_{\sigma_\ell,t}, \varepsilon_{\theta,t}, \varepsilon_{\sigma_{\theta},t} \sim \mathcal{N} (0, 1) \)
Approaches to Uncertainty Shocks in the Literature

- MIT aggregate shocks
- Low-order perturbation
- Reduced state space approximations

We address these problems with AI and deep learning (DL)

- Aggregate shocks in the solution procedure
- Global nonlinear solutions
- True state space
1. **HANK model**: \[
\begin{align*}
E_\epsilon [f_1 (X (s), \epsilon)] &= 0 \\
& \vdots \\
E_\epsilon [f_n (X (s), \epsilon)] &= 0
\end{align*}
\]

$s = \text{state}$, $X (s) = \text{decision function}$, $\epsilon = \text{innovations}$.

2. Parameterize $X (s) \approx \varphi (s; \theta)$ with a **deep neural network**.

3. Construct **objective function** for DL training

\[
\min_\theta (E_\epsilon [f_1 (\varphi (s; \theta), \epsilon)]]^2 + \ldots + (E_\epsilon [f_n (\varphi (s; \theta), \epsilon)])^2 \to 0
\]

4. **All-in-one expectation** operator is a critical novelty:

\[
(E_\epsilon [f_j (\varphi (s; \theta), \epsilon)])^2 = E_{(\epsilon_1, \epsilon_2)} [f_j (\varphi (s; \theta), \epsilon_1) \cdot f_j (\varphi (s; \theta), \epsilon_2)]
\]

with $\epsilon_1, \epsilon_2 = \text{two independent draws}$.

4. **Stochastic gradient descent** for training (random grids)

5. Google **TensorFlow** platform — software that leads to break-ground applications (image, speech recognition, etc).
Krusell and Smith (1998) versus the Present Paper

- **Krusell and Smith** (1998) use a reduced state space: 
  \( X_i \) (variables of agent \( i \), aggregate moments) 
  \( \Rightarrow \) few state variables

- **The present paper** uses the true state space: 
  \( X_i \) (variables of all agents, distributions) 
  \( \Rightarrow \) hundreds of state variables

*How do we deal with such a large state space?*

1. Neural network automatically performs the model reduction 
   – it learns to summarize information from many inputs into a smaller set of hidden layers.

2. Neural network deals with ill conditioning 
   – it learns to ignore collinear and redundant variables.
Individual Policy Rules

- $\eta_\ell$: individual idiosyncratic productivity

<table>
<thead>
<tr>
<th>State variable</th>
<th>$R$</th>
<th>$b$</th>
<th>$s$</th>
<th>$\eta_R$</th>
<th>$\eta_\theta$</th>
<th>$\sigma_\ell$</th>
<th>$\sigma_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>$R^*$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>$\ln \sigma_\ell^*$</td>
<td>$\ln \sigma_\theta^*$</td>
</tr>
</tbody>
</table>
Correlation of Wealth Gini with Different Shocks

<table>
<thead>
<tr>
<th>Shock</th>
<th>Level</th>
<th>Stochastic Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TFP</td>
<td>Monetary policy</td>
</tr>
<tr>
<td>( \text{corr}(Gini (liquid), shock) )</td>
<td>0.1067</td>
<td>-0.0306</td>
</tr>
<tr>
<td>( \text{corr}(Gini (illiquid), shock) )</td>
<td>0.0651</td>
<td>0.0031</td>
</tr>
</tbody>
</table>
Aggregate Impulse Responses to Aggregate Uncertainty

- **Machines**: Graph showing the response of machines over time.
- **Output**: Graph showing the response of output over time.
- **Inflation**: Graph showing the response of inflation over time.
- **Consumption**: Graph showing the response of consumption over time.
- **Agg.Uncertainty**: Graph showing the response of aggregate uncertainty over time.
- **Wealth Gini**: Graph showing the response of wealth Gini over time.
Conclusion

• Solution framework suited for studying shocks to higher order moments
• Preliminary results are promising
• Looking forward to comments
Thank you!
Household Problem

\[
\max_{c_t, \ell_t, i_t, s_t, b_t, k_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t(j)^{1-\gamma} - 1}{1 - \gamma} + \psi \frac{(1 - \ell_t(j))^{1-\phi} - 1}{1 - \phi} \right].
\]

s.t. \(c_t(j) + i_t(j) + b_t(j) + \Psi(i_t(j), s_{t-1}(j), k_{t-1}(j)) = \frac{R_{t-1}}{\pi_t} b_{t-1}(j) + w_{t} \ell_t(j) \exp(\eta_{\ell, t}(j)) + \tau_t(j),\)

\[
Q_t s_t(j) + k_t(j) = \left[1 - d + r_t^k\right] k_{t-1}(j) + i_t(j) + [Q_t + (1 - \delta) \Pi_t] s_{t-1}(j),
\]

\[
Q_t s_t(j) \geq 0, \quad k_t(j) \geq 0, \quad b_t(j) \geq -\bar{b},
\]

- \(\Psi(\cdot, \cdot)\): adjustment cost on machines and shares
- \(\tau_t(j)\): transfers
Savings Decisions and Transfers

- Liquid assets: \( b_t(j) \)
- Illiquid assets: \( s_t(j), k_{t-1}(j) \)

\[
\Psi (i_t(j), s_{t-1}(j), k_{t-1}(j)) = \]

\[
\Gamma_1 |i_t(j)| +
\frac{\Gamma_2}{\Gamma_3} \left( \frac{|i_t(j)|}{[Q_t + (1 - \delta) \Pi_t] s_{t-1}(j) + [1 - d + r^k_t] k_{t-1}(j) + \varepsilon - \xi} \right)
\]

\[
\left( [Q_t + (1 - \delta) \Pi_t] s_{t-1}(j) + [1 - d + r^k_t] k_{t-1}(j) + \varepsilon \right),
\]

- Transfers

\[
\tau_t(j) = \delta \Pi_t + \delta (\ell_t(j) \exp(\eta_{\ell,t}(j)) - H_t)
\]

\[
\int \tau_t(j) \, dj = \delta \Pi
\]
Firms

- **Production function**

\[ Y_t(i) = \exp(\eta_{\theta,t}) K_{t-1}(i)^\alpha H_t(i)^{1-\alpha} \]

- **Price adjustment cost**

\[ \xi(P_t(i), P_{t-1}(i)) = \frac{\mu}{2\kappa_p (\mu - 1) \pi^*} \left( \log \left( \frac{P_t(i)}{P_{t-1}(i) \frac{1}{\pi^*}} \right) \right)^2 \]

- **Aggregate profits**

\[ \Pi_t = Y_t - r_t^k K_{t-1} - w_t H_t \]

\[ Y_t = \exp(\eta_{\theta,t}) K_{t-1}^\alpha H_t^{1-\alpha} \left\{ 1 - \frac{\mu}{2\kappa_p (\mu - 1) \pi^*} \left( \log \left( \frac{\pi_t}{\pi^*} \right) \right)^2 \right\} \]
Central Bank

• Taylor rule subject to ZLB

\[ R_t \equiv \max \left\{ 1.0, R_\ast \left( \frac{R_{t-1}}{R_\ast} \right) ^\mu \left[ \left( \frac{\pi_t}{\pi_\ast} \right)^{\phi_\pi} \left( \frac{Y_t}{Y_\ast} \right)^{\phi_y} \right]^{1-\mu} \exp(\eta_{R,t}) \right\} \]

• Monetary policy shock

\[ \eta_{R,t} = \rho^R \eta_{R,t-1} + \sigma_R \varepsilon_{R,t}, \quad \varepsilon_{R,t} \sim \mathcal{N}(0, 1) \]
Market Clearing

- Market clearing
  \[ \int_0^1 s_t(j) \, dj = 1 \]
  \[ \int_0^1 b_t(j) \, dj = 0 \]
  \[ C_t + K_t - (1 - d) K_{t-1} + \int_0^1 AC(i) \, di = Y_t \]
- \[ \int_0^1 AC(i) \, di = \int_0^1 \Psi(i_t(j), Q_t s_{t-1}(j) + k_{t-1}(j)) \, dj \] : aggregate cost of adjustment
- \[ C_t = \int_0^1 c_t(j) \, dj \]
- \[ K_{t-1} = \int_0^1 k_{t-1}(j) \, dj \]