Household Savings and Monetary Policy under Individual and Aggregate Stochastic Volatility

Yuriy Gorodnichenko,[1] Lilia Maliar,[2] Serguei Maliar,[3] & Christopher Naubert[4]

[1]University of California at Berkeley and NBER; [2]CUNY Graduate Center and CEPR; [3]Santa Clara University;

[4]CUNY Graduate Center

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Motivation

- Four major developments in macroeconomics:
  – heterogeneity in income or productivity and assets with differing liquidity (machine, shares, liquid bonds)
  – aggregate (and idiosyncratic) uncertainty done right
  – global solutions
  – redistribution
- These are usually done in isolation.

This paper: We do all four in one framework

HANK Model

- Households
  - Three types of assets: bonds (liquid), shares and machines (illiquid)
  - Three borrowing constraints, one for each asset
  - Idiosyncratic shocks to productivity level and volatility
  - Heterogenous labor
- Firms
  - CRS with machines and labor
  - Aggregate shocks to TFP level and volatility
  - Sticky prices (Rotemberg)
- Government
  - Fiscal policy (lump-sum taxes, for now)
  - Monetary policy (Taylor rule with ZLB)

Levels and Uncertainty Shocks

- Household productivity

$$\text{risk:} \quad \eta_{\ell,t}(j) = \rho^{\ell}\eta_{\ell,t-1}(j) + \exp\left(\sigma_{\ell,t-1}\right)\varepsilon_{\ell,t}(j)$$
$$\text{uncertainty:} \quad \sigma_{\ell,t} = \rho^{\sigma_l}\sigma_{\ell,t-1} + \sigma_{\sigma_\ell}\varepsilon_{\sigma_\ell,t}$$

- Aggregate TFP

$$\text{risk:} \quad \eta_{\theta,t} = \rho^{\theta}\eta_{\theta,t-1} + \exp\left(\sigma_{\theta,t-1}\right)\varepsilon_{\theta,t}$$
$$\text{uncertainty:} \quad \sigma_{\theta,t} = \rho^{\sigma_\theta}\sigma_{\theta,t-1} + \sigma_{\sigma_\theta}\varepsilon_{\sigma_\theta,t}$$

where $\varepsilon_{\ell,t}$, $\varepsilon_{\sigma_\ell,t}$, $\varepsilon_{\theta,t}$, $\varepsilon_{\sigma_\theta,t} \sim \mathcal{N}(0,1)$

Approaches to Uncertainty Shocks in the Literature

- MIT aggregate shocks
- Low-order perturbation
- Reduced state space approximations

We address these problems with AI and deep learning (DL)

- Aggregate shocks in the solution procedure
- Global nonlinear solutions
- True state space

Deep Learning Analysis of Maliar, Maliar, Winant (2019)

1. **HANK model**: $\begin{cases} E_\epsilon\left[f_1\left(X\left(s\right),\epsilon\right)\right]=0 \\ \ldots \\ E_\epsilon\left[f_n\left(X\left(s\right),\epsilon\right)\right]=0 \end{cases}$

   s = state, $X\left(s\right)$ = decision function, ϵ = innovations.
2. Parameterize $X\left(s\right)\simeq\varphi\left(s;\theta\right)$ with a **deep neural network**.
3. Construct **objective function** for DL training

$$\min_\theta\left(E_\epsilon\left[f_1\left(\varphi\left(s;\theta\right),\epsilon\right)\right]\right)^2+\ldots+\left(E_\epsilon\left[f_n\left(\varphi\left(s;\theta\right),\epsilon\right)\right]\right)^2\to 0$$

4. **All-in-one expectation** operator is a critical novelty:

$$\left(E_\epsilon\left[f_j\left(\varphi\left(s;\theta\right),\epsilon\right)\right]\right)^2=E_{(\epsilon_1,\epsilon_2)}\left[f_j\left(\varphi\left(s;\theta\right),\epsilon_1\right)\cdot f_j\left(\varphi\left(s;\theta\right),\epsilon_2\right)\right]$$

   with ϵ_1,ϵ_2 = two independent draws.
4. **Stochastic gradient descent** for training (random grids)
5. Google **TensorFlow** platform – software that leads to break-ground applications (image, speach recongnition, etc).

Krusell and Smith (1998) versus the Present Paper

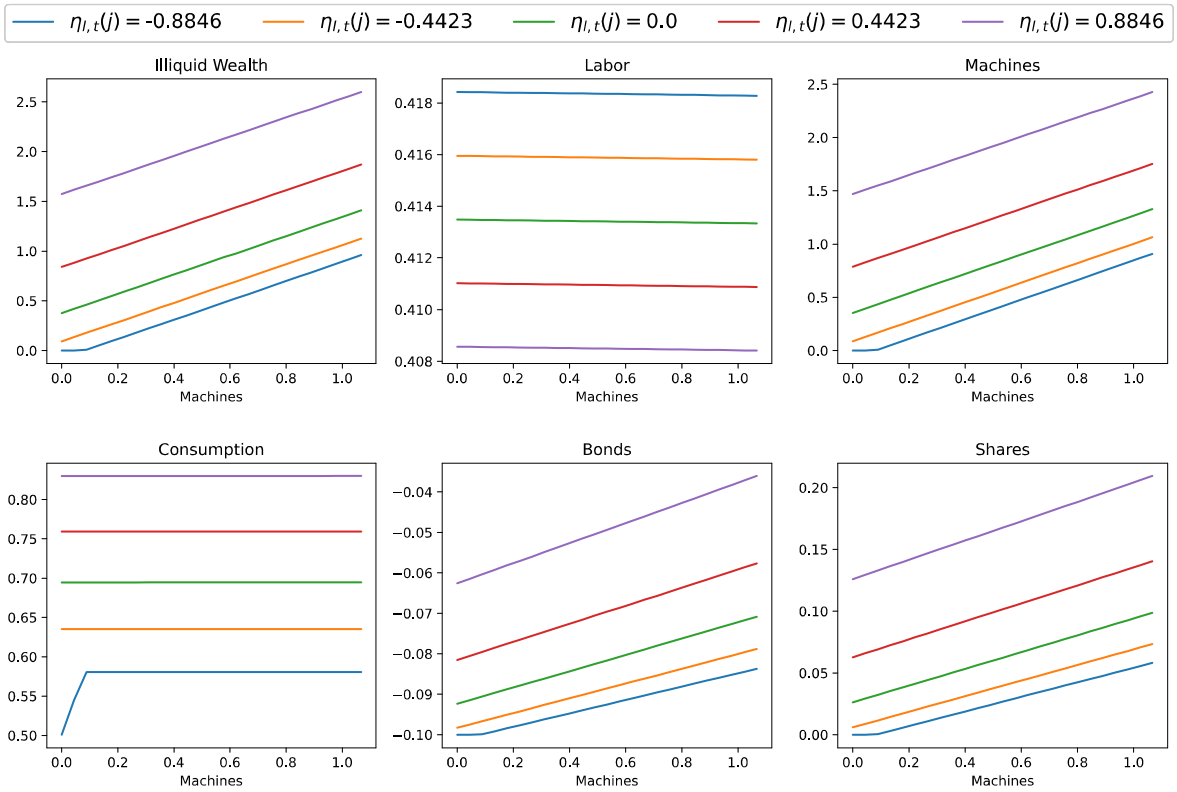
- **Krusell and Smith** (1998) use a reduced state space:
  X_i (*variables of agent* i, *aggregate moments*)
  $\Rightarrow$ few state variables
- **The present paper** uses the true state space:
  X_i (*variables of all agents*, *distributions*)
  $\Rightarrow$ hundreds of state variables

How do we deal with such a large state space?

1. Neural network automatically performs the model reduction
   – it learns to summarize information from many inputs into a smaller set of hidden layers.
2. Neural network deals with ill conditioning
   – it learns to ignore collinear and redundant variables.

Individual Policy Rules

- η_ℓ: individual idiosyncratic productivity

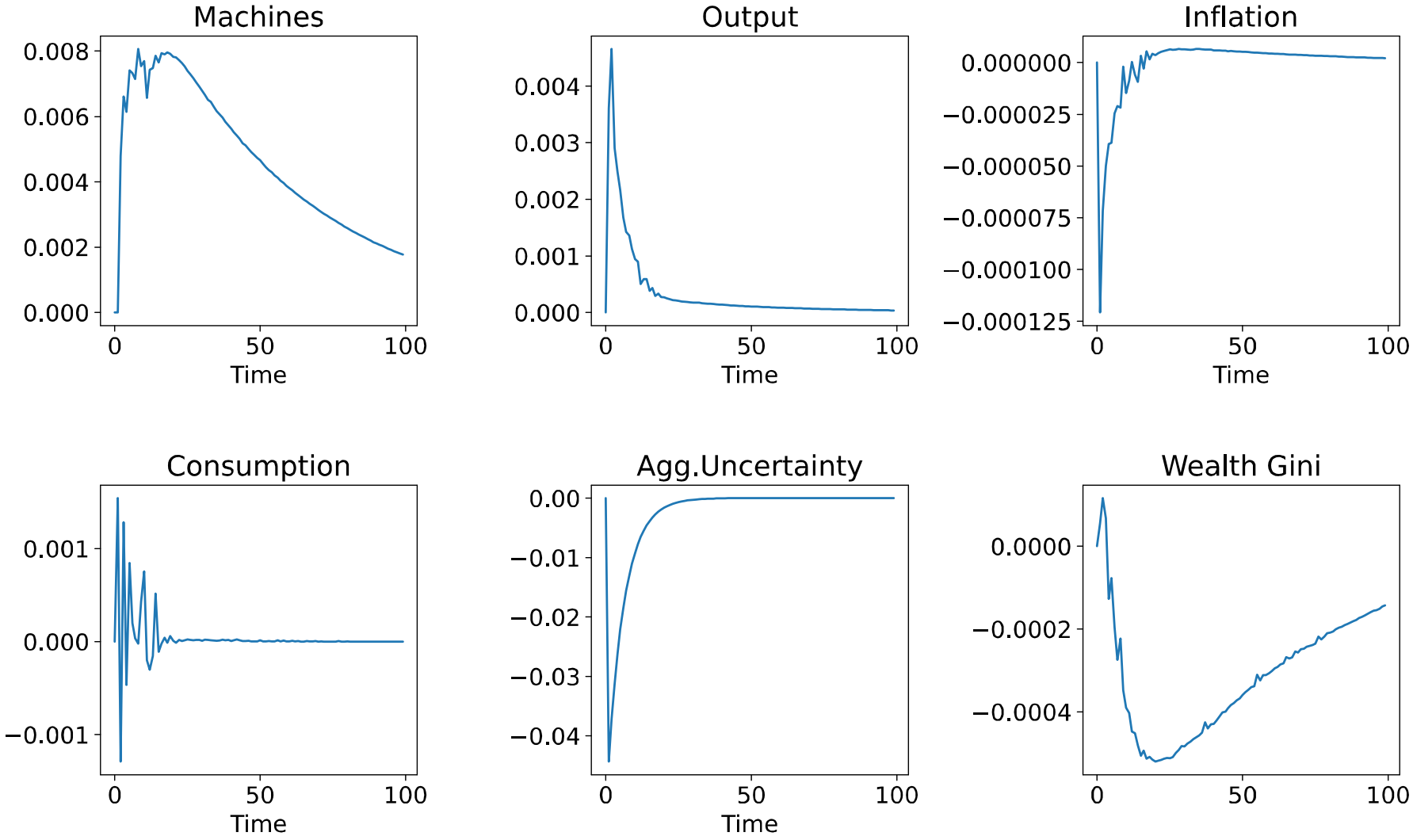


State variable	R	b	s	η_R	η_θ	σ_ℓ	σ_θ
Value	R^*	0.0	0.0	0.0	0.0	$\ln \sigma_\ell^*$	$\ln \sigma_\theta^*$

Correlation of Wealth Gini with Different Shocks

Shock	Level		Stochastic Volatility	
	TFP	Monetary policy	Idiosync.	TFP
$corr(Gini\,(liquid)\,, shock)$	0.1067	-0.0306	0.0216	0.0073
$corr(Gini\,(illiquid)\,, shock)$	0.0651	0.0031	-0.0255	0.0487

Aggregate Impulse Responses to Aggregate Uncertainty



Conclusion

- Solution framework suited for studying shocks to higher order moments
- Preliminary results are promising
- Looking forward to comments

Thank you!

Household Problem

$$\max_{c_t,\ell_t,i_t,s_t,\widetilde{b}_t,k_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t(j)^{1-\gamma} - 1}{1-\gamma} + \psi \frac{(1-\ell_t(j))^{1-\vartheta} - 1}{1-\vartheta} \right]$$

$$\text{s.t. } c_t(j) + i_t(j) + b_t(j) + \Psi(i_t(j), s_{t-1}(j), k_{t-1}(j)) =$$

$$\frac{R_{t-1}}{\pi_t} b_{t-1}(j) + w_t \ell_t(j) \exp\left(\eta_{\ell,t}(j)\right) + \tau_t(j),$$

$$Q_t s_t(j) + k_t(j) =$$

$$\left[1 - d + r_t^k\right] k_{t-1}(j) + i_t(j) + \left[Q_t + (1-\delta)\Pi_t\right] s_{t-1}(j),$$

$$Q_t s_t(j) \geq 0, \quad k_t(j) \geq 0, \quad b_t(j) \geq -\overline{b},$$

- $\Psi(\cdot,\cdot)$: adjustment cost on machines and shares
- $\tau_t(j)$: transfers

Savings Decisions and Transfers

- Liquid assets: $b_t(j)$
- Illiquid assets: $s_t(j)$, $k_{t-1}(j)$

$$\Psi\left(i_t(j), s_{t-1}(j), k_{t-1}(j)\right) =$$

$$\Gamma_1 \left|i_t(j)\right| +$$
$$\frac{\Gamma_2}{\Gamma_3}\left(\frac{\left|i_t(j)\right|}{\left[Q_t + (1-\delta)\Pi_t\right] s_{t-1}(j) + \left[1 - d + r_t^k\right] k_{t-1}(j) + \varepsilon} - \xi\right)$$
$$\left(\left[Q_t + (1-\delta)\Pi_t\right] s_{t-1}(j) + \left[1 - d + r_t^k\right] k_{t-1}(j) + \varepsilon\right),$$

- Transfers

$$\tau_t(j) = \delta\Pi_t + \delta\left(\ell_t(j)\exp\left(\eta_{\ell,t}(j)\right) - H_t\right)$$
$$\int \tau_t(j)\, dj = \delta\Pi$$

Firms

- Production function

$$Y_t(i) = \exp\left(\eta_{\theta,t}\right) K_{t-1}(i)^{\alpha} H_t(i)^{1-\alpha}$$

- Price adjustment cost

$$\xi\left(P_t(i), P_{t-1}(i)\right) \equiv \frac{\mu}{2\kappa^p(\mu-1)\pi^*}\left(\log\left(\frac{P_t(i)}{P_{t-1}(i)}\frac{1}{\pi^*}\right)\right)^2$$

- Aggregate profits

$$\Pi_t = Y_t - r_t^k K_{t-1} - w_t H_t$$

$$Y_t = \exp\left(\eta_{\theta,t}\right) K_{t-1}^{\alpha} H_t^{1-\alpha}\left\{1 - \frac{\mu}{2\kappa^p(\mu-1)\pi^*}\left(\log\left(\frac{\pi_t}{\pi^*}\right)\right)^2\right\}$$

Central Bank

- Taylor rule subject to ZLB

$$R_t \equiv \max\left\{1.0, R_* \left(\frac{R_{t-1}}{R_*}\right)^{\mu} \left[\left(\frac{\pi_t}{\pi_*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_*}\right)^{\phi_y}\right]^{1-\mu} \exp\left(\eta_{R,t}\right)\right\}$$

- Monetary policy shock

$$\eta_{R,t} = \rho^R \eta_{R,t-1} + \sigma_R \varepsilon_{R,t}, \qquad \varepsilon_{R,t} \sim \mathcal{N}(0,1)$$

Market Clearing

- Market clearing

$$\int_0^1 s_t(j)\,dj = 1$$

$$\int_0^1 b_t(j)\,dj = 0$$

$$C_t + K_t - (1-d)\,K_{t-1} + \int_0^1 \mathrm{AC}(i)\,di = Y_t$$

- $\int_0^1 \mathrm{AC}(i)\,di = \int_0^1 \Psi\left(i_t(j), Q_t s_{t-1}(j) + k_{t-1}(j)\right) dj$: aggregate cost of adjustment
- $C_t = \int_0^1 c_t(j)\,dj$
- $K_{t-1} = \int_0^1 k_{t-1}(j)\,dj$