Special Topics in Macro Theory
Computational Methods for Economists
Econ-81500

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The City University of New York
The Graduate Center

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Lectures: Thursday, 2:00 pm - 4:00 pm.
Office hours: Wednesday, 13:00 pm – 14:00 pm.

Course Description

Overview
This course studies computational approaches for solving dynamic economic models. The objectives of the course are threefold.
1. It provides background in numerical analysis (approximation, integration, optimization, error analysis), and describes local and global numerical methods (perturbation, Smolyak, endogenous grid, stochastic simulation, cluster grid methods).
2. It shows applications from recent economic literature representing challenges to computational methods (new Keynesian models with a zero lower bound, default risk models, Krusell-Smith models, international trade models, overlapping-generations models, nonstationary growth models, dynamic games).
3. It surveys recent developments in software and hardware (Python, Julia, GPUs, parallel computing, supercomputers), as well as machine learning techniques.

Learning goals and outcomes
1) Demonstrate good understanding of the existing numerical techniques used for solving dynamic economic problems.
2) Develop good programming skills.
3) Demonstrate the ability to build a solution algorithm suitable for the given economic problem.
4) Demonstrate the ability to find new research questions in light of existing theories.
5) Demonstrate the ability to design a theoretical model aimed at answering potentially interesting research questions in the field.
6) Develop writing skills consistent with the requirement of professional publications.

Assessment
The course grade will be based on
1. Individual problem sets (40%). These problem sets provide opportunities to bring lecture material into practice. This assignment relates to learning goals 1), 2) and 3).
2. A group project (20%). Each group will study and present in class an existing computational technique. This relates to learning goal 1) and 2).
3. A final individual project (40%). Each student will write a short research paper in which computational methods are used to address some relevant economic questions and will present the results in class. This assignment relates to learning goals 4), 5), and 6).

ACTIVITIES	PERCENTAGES
Problem Sets	40%
Group Project	20%
Final Individual Project	40%

Prerequisites
It is best if students are familiar with basic dynamic optimization theory, and have a good undergraduate background in mathematics. However, any student who is interested in the course and has passed the first-year core econometrics, microeconomics, and macroeconomics courses will be able to follow the course.

Computing Languages
Students will need to know some computational language. Our suggestion is to learn MATLAB since it is easy to quickly learn, has nice graphical tools, and is commonly used in the economic literature. If you do not know MATLAB, you can consult any one of many online tutorials. Do a google search for "MATLAB tutorial" and you will find links to many useful tutorials. It is also useful to learn MATLAB even if you already know other programming languages. Other kinds of software may be used to solve the exercises (C, Fortran, Python, Julia). Finally, for parallel computing, we will use MATLAB, C and C++.

Course Schedule
Note: *The content is subject to changes depending on the student's progress and feedback.*

INTRODUCTION

1. Introduction to the course.
Richer and more complex economic models. What can we solve analytically? Challenges of economic dynamics. Estimation of the models. Methodology. Objectives of the course. Outline of the course. Review of the topics.

2. Outline and key ingredients of global solution methods.
Neoclassical growth model. Bellman equation. Euler equation. An example of a global solution method for neoclassical growth model.

PART I. BACKGROUND IN NUMERICAL ANALYSIS WITH AN APPLICATION TO THE ONE-AGENT GROWTH MODEL

1. Approximation and interpolation of functions.
General idea. Interpolation, approximation, and extrapolation. Lagrange interpolation. Hermite

interpolation. Orthogonal polynomials. Chebyshev regression. Splines. Complete polynomials.

2. Deterministic and Monte Carlo integration for low-dimensional problems.
Newton-Cotes integration formulas, Monte Carlo and Gaussian integration methods.

3. Linear equations and numerically stable methods.
Cholesky decomposition, condition numbers, Gauss-Jacobi and Gauss-Seidel methods.
SVD, LAD, regularization, principle component analysis

4. Nonlinear equations. Solvers.
Search methods, bisection, Newton method, BFGS and DFP updates. Gauss-Jacobi, Gauss-Seidel, Newton, continuation and homotopy methods. Fitting methods.

5. Optimization. The importance of an iterative scheme. ECM and EGM methods.
Endogenous grid method (EGM) and envelope condition method (ECM)

6. **Projection methods for differential, integral, and functional equations**
Projection methods for solving ordinary differential equations, as well as the more complex equations arising in dynamic economic models.

7. Convergence rates and accuracy measures
Convergence rates. Residuals in the model's equations. Errors in model's variables. Economically meaningful measures of accuracy.

PART II. NUMERICAL ANALYSIS FOR HIGH DIMENSIONAL APPLICATIONS (WITH AN EXAMPLE OF A MULTI-AGENT GROWTH MODEL)

1. Monomial and quasi-Monte Carlo integration methods
Nonproduct deterministic integration methods, random and quasi-random sequences.

2. Smolyak technique
Smolyak grid, Smolyak polynomials, Lagrange interpolation.

3. Epsilon-distinguishable set and cluster grid techniques.
Essentially ergodic-set methods.

4. Precomputation of intertemporal-choice functions and integrals.
Multivariate continuous-state and discrete-state expectations.

PART III. LOCAL SOLUTION METHODS

1. Linearization and perturbation
Log-linearization in a deterministic model. Log-linearization in a stochastic model. Perturbation.

2. Recent developments of perturbation methods

Change of variables. A hybrid of local and global solutions. Risky steady state. Penalty functions. Borrowing constraints. Regime switches. Pruning methods.

3. Perturbation software
Dynare automated software. Other packages.

PART IV. CHALLENGING ECONOMIC APPLICATIONS IN THE LITERATURE

1. Nonstationary and unbalanced growth models with infinite horizon
Models with time-varying parameters (parameter shifts and drifts), seasonal changes, stochastic and deterministic changes in volatility. Unbalanced growth models.

2. OLG models
A large-scale OLG model.

3. New Keynesian models
Stylized new Keynesian models. Taylor rule and a zero lower bound on nominal interest rates. Large-scale central banking models.

4. Models with a continuum of state variables
Approximating kinks in policy functions and Kuhn-Tucker conditions. Aiyagari model. Krusell-Smith analysis.

5. A model with default risk.
Application of the envelope condition method to a model with sovereign default.

6. Dynamic games
Markov perfect equilibria, stochastic games and time inconsistency.

7. Applications of machine learning techniques to economics
Supervised learning. Unsupervised learning. A deep learning algorithm for solving economic models.

PART V. HARDWARE AND SOFTWARE

1. MATLAB, Python and Julia: what to choose in economics?
Language comparisons with economic applications

2. Hardware and software for computationally intense problems
Parallel computing with MATLAB and C++. GPU. Clusters and supercomputers.

Main texts:

Kenneth L. Judd, (1998). *Numerical Methods in Economics*. The MIT Press.

Lilia Maliar and Serguei Maliar, (2014). "Numerical Methods for Large Scale Dynamic Economic Models" in: Schmedders, K. and K. Judd (Eds.), *Handbook of Computational Economics*, Volume 3, Chapter 7, Amsterdam: Elsevier Science.

Supplementary texts:

Jerome Adda and Russell W. Cooper, (2003). *Dynamic Economics: Quantitative Methods and Applications*. The MIT Press.

Paolo Brandimarte, (2006). *Numerical Methods in Finance and Economics*. John Weley&Sons.
Burkhard Heer and Alfred Maussner, (2004). *Dynamic General Equilibrium Modeling*. Springer.

Kenneth L. Judd, Lilia Maliar and Serguei Maliar, (2017). *Ergodic set methods for solving dynamic economic models*. Under the contract with the MIT Press, in progress.

Jianjun Miao, *Economic Dynamics in Discrete Time*, (2014). The MIT Press.

Mario J. Miranda and Paul L. Fackler, (2002). *Applied Computational Economics and Finance*. The MIT Press.

G. C. Lim and Paul D. McNelis, (2008). *Computational Macroeconomics for the Open Economy*. The MIT Press.

David A. Kendrick, P. Ruben Mercado, Hans M. Amman, (2006). *Computational Economics*. Princeton University Press.

John Stachurski, (2009). Economic Dynamics. *Theory and Computation*. The MIT Press.

Reading for Part I

Arellano, C., L. Maliar, S. Maliar and V. Tsyrennikov, (2016). Envelope condition method with an application to default models. Journal of Economic Dynamics and Control 69*,* 436-459*.*

Barillas, F. and J. Fernández-Villaverde, (2007). A generalization of the endogenous grid method. Journal of Economic Dynamics and Control, Elsevier 31, 2698-2712.

Carroll, K., (2005). The method of endogenous grid points for solving dynamic stochastic optimal problems. Economic Letters 91, 312-320.

Judd, K., Maliar, L. and S. Maliar (2011). Numerically stable and accurate stochastic simulation approaches for solving dynamic economic models. Quantitative Economics 22, 173-210.

Judd, K. (1998). Numerical Methods in Economics. The MIT Press.

Judd, K., L. Maliar and S. Maliar, (2017). Lower bounds on approximation errors to numerical solutions of dynamic economic models. Econometrica 85 (3), 991-1020.

Maliar L. and S. Maliar, (2013). Envelope condition method versus endogenous grid method for solving dynamic programming problems. Economic Letters 120, 262-266.

Maliar, L. and S. Maliar, (2014). "Numerical Methods for Large Scale Dynamic Economic Models" in: Schmedders, K. and K. Judd (Eds.), Handbook of Computational Economics, Volume 3, Chapter 7, Amsterdam: Elsevier Science.

Marcet, A., and G. Lorenzoni (1999). The parameterized expectation approach: some practical issues. In: R. Marimon and A. Scott (Eds.) Computational Methods for Study of Dynamic Economies. Oxford University Press, New York, pp. 143-171.

Reading for Part II, Section 1

Judd, K., Maliar, L. and S. Maliar (2011). Numerically stable and accurate stochastic simulation approaches for solving dynamic economic models. Quantitative Economics 22, 173-210.

Judd, K. (1998). Numerical Methods in Economics. The MIT Press.

Rust, J., (1997). Using randomization to break the curse of dimensionality. Econometrica 65, 487-516.

Stroud A. (1971). Approximate Integration of Multiple Integrals. Prentice Hall: Englewood Cliffs, New Jersey.

Reading for Part II, Section 2

Brumm, J., Scheidegger, S. (2017). Using adaptive sparse grids to solve high-dimensional dynamic models, Econometrica, forthcoming-

Judd, K, Maliar, L., Maliar, S. and R. Valero, (2014). Smolyak method for solving dynamic economic models: Lagrange interpolation, anisotropic grid and adaptive domain, Journal of Economic Dynamics and Control 44, 92-103.

Krueger, D. and F. Kubler, (2004). Computing equilibrium in OLG models with production. Journal of Economic Dynamics and Control 28, 1411-1436.

Malin, B., Krueger, D., and F. Kubler, (2011). Solving the multi-country real business cycle model using a Smolyak-collocation method. Journal of Economic Dynamics and Control 35 (2), 229-239.

Smolyak, S., (1963). Quadrature and interpolation formulas for tensor products of certain classes of functions. Soviet Mathematics, Doklady 4, 240-243.

Reading for Part II, Section 3

Eldén, L. (2007), Matrix Methods in Data Mining and Pattern Recognition. SIAM, Philadelphia.

Hastie, T., R. Tibshirani, and J. Friedman, (2009). The Elements of Statistical Learning. Springer.

Judd, K., Maliar, L. and S. Maliar, (2010). A cluster-grid projection algorithm: solving problems with high dimensionality, NBER 15965.

Maliar, S., Maliar, L., and K. Judd, (2011). Solving the multi-country real business cycle model using ergodic set methods, Journal of Economic Dynamics and Control 35(2), 207-228.

Maliar, L., and S. Maliar, (2015). Merging simulation and projection approaches to solve high-dimensional problems with an application to a new Keynesian model, Quantitative Economics 6, 1-47.

Niederreiter, H., (1992). Random Number Generation and Quasi-Monte Carlo Methods. Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania.

Temlyakov, V., (2011). Greedy approximation. Cambridge University Press, Cambridge.

Reading for Part II, Section 4

Judd, K., L. Malia, S. Malia and I. Tsener, (2017). How to solve dynamic stochastic models computing expectations just once. Quantitative Economics, 8(3), 851-893.

Maliar, L. and S. Maliar, (2005). Solving nonlinear stochastic growth models: an algorithm computing value function by simulations", *Economics Letters* 87, 135-140.

Reading for Part III

Adjemian, S., Bastani, H., Juillard, M., Mihoubi, F., Perendia, G., Ratto, M. and S. Villemot, (2011). Dynare: reference manual, version 4. Dynare Working Papers 1, CEPREMAP.

Aruoba, S. B., Fernández-Villaverde, J. and J. Rubio-Ramírez, (2006). Comparing solution methods for dynamic equilibrium economies. Journal of Economic Dynamics and Control 30, 2477-2508.

Den Haan and J. De Wind, (2012). Nonlinear and stable perturbation-based approximations. Journal of Economic Dynamics and Control 36, 1477-1497.

Fernández-Villaverde, J. and J. Rubio-Ramírez, (2006). Solving DSGE models with perturbation methods and a change of variables, Journal of Economic Dynamics and Control 30, 2509—2531

Judd, K., (2003). Perturbation methods with nonlinear changes of variables. Manuscript.

Maliar, L., Maliar, S., and S. Villemot, (2013). Taking perturbation to the accuracy frontier: a hybrid of local and global solutions, Computational Economics 42(3), 307-325.

Schmitt-Grohé S. and M. Uribe, (2004). Solving dynamic general equilibrium models using a second-order approximation to the policy function. Journal of Economic Dynamics and Control 28(4), 755-775.

Readings for Part IV, Section 1

Adjemian, S. and M. Juillard, (2013). Stochastic extended path approach. Manuscript.

Fair, R. and J. Taylor, (1983). Solution and maximum likelihood estimation of dynamic nonlinear rational expectations models. Econometrica 51, 1169-1185.

Schmitt-Grohé, S. and M. Uribe, (2012). What's news in business cycles? Econometrica 80, 2733-2764.

Lepetyuk, V., L. Maliar, S. Maliar and J. B. Taylor, (2019). Extended function path perturbation for nonstationary and unbalanced growth models. Manuscript.

Maliar, L., S. Maliar, J. Taylor and I. Tsener, (2015). A tractable framework for analyzing a class of nonstationary Markov Models, NBER 21155.

Maliar, L., Maliar S. and I. Tsener, (2018). Capital-skill complementarity: twenty years after. Manuscript.

Reading for Part IV, Section 2

Hasanhodzic, J. and L. Kotlikoff, L., (2013). Generational risk–is it a big deal? Simulating an 80-period OLG model with aggregate s hocks. NBER 19179.

Juillard, M. and S. Villemot, (2011). Multi-country real business cycle models: Accuracy tests and testing bench. Journal of Economic Dynamics and Control 35 (2), 178-185.

Kollmann, R., Kim, S., and J. Kim, (2011). Solving the multi-country real business cycle model using a perturbation method, Journal of Economic Dynamics and Control 35 (2), 203-206.

Kollmann, R., Maliar, S., Malin, B. and P. Pichler, (2011). Comparison of solutions to the multi-country real business cycle model, Journal of Economic Dynamics and Control 35(2), 186-202.

Lee, S., L. Kotlikoff, L. Maliar, and S. Maliar, (2019). Long-term implications of aging population in the macroeconomy, Manuscript.

Maliar, S., Maliar, L. and K. Judd, (2011). Solving the multi-country real business cycle model using ergodic set methods, Journal of Economic Dynamic and Control 35(2), 207-228.

Maliar, L., Maliar, S., and S. Villemot, (2013). Taking perturbation to the accuracy frontier: a hybrid of local and global solutions, Computational Economics 42(3), 307-325.

Reading for Part IV, Section 3

Aruoba, S., Cuba-Borda, P., and F. Schorfheide, (2017). Macroeconomic dynamics near the ZLB: a tale of two countries, The Review of Economic Studies 85 (1), 87-118.

Christiano, L., M. Eichenbaum, and S. Rebelo, (2011). When is the government spending multiplier large? Journal of Political Economy 119(1), 78-121.

Fernández-Villaverde, J., Gordon, G., Guerrón-Quintana, P. and J. Rubio-Ramírez, (2012). Nonlinear adventures at the zero lower bound, NBER 18058.

Fernández-Villaverde, J., G. Gordon, P. Guerrón-Quintana, and J. Rubio-Ramírez, (2015). Nonlinear adventures at the zero lower bound. Journal of Economic Dynamics and Control, 182-204.

Lepetuyk, V., L. Maliar and S. Maliar (2019). When the U.S. catches a cold, Canada sneezes: a lower-bound tale told by deep learning. CEPR working paper DP 14025.

Maliar, L., and S. Maliar, (2014). Merging simulation and projection approaches to solve high-dimensional problems with an application to a new Keynesian model, Quantitative Economics 6, 1-47.

Mertens, K. and M. Ravn, (2011). Credit channels in a liquidity trap. CEPR discussion paper 8322.

Smets, F. and R. Wouters, (2007). Shocks and frictions in US business cycles: a Bayesian DSGE approach. American Economic Review 97 (3), 586-606.

Reading for Part IV, Section 4

Ahn, S., G. Kaplan, B. Moll, T. Winberry, C. Wolf, (2017). When Inequality matters for macro and macro matters for inequality. Manuscript.

Auclert, A., B. Bardóczy, M. Rognlie, L. Straub, (2019). Using the sequence-space Jacobian to solve and estimate heterogeneous-agent models. Manuscript.

Bayer, C. and R. Lutticke, (2018). Solving heterogeneous agent models with aggregate uncertainty and many idiosyncratic states in discrete time by perturbation methods. Manuscript.

Boppart, T., Krusell, P. and K. Mitman, (2018). Exploiting MIT shocks in heterogeneous-agent: The impulse-response as a numerical derivative. Journal of Economic Dynamics and Control 89, 68-92.

Den Haan, (2010a). Assessing the accuracy of the aggregate law of motion in models with heterogeneous agents, Journal of Economic Dynamics and Control 34, 79-99.

Den Haan, (2010b) Comparison of solutions to the incomplete markets model with aggregate uncertainty, Journal of Economic Dynamics and Control 34, 4-27.

Kim, S., R. Kollmann, and J. Kim, (2010). Solving the incomplete markets model with aggregate uncertainty using a perturbation method, Journal of Economic Dynamics and Control 34, 50-58.

Krusell, P., and A. Smith, (1998). Income and wealth heterogeneity in the macroeconomy. Journal of Political Economy 106:5, 868-96.

Ljungqvist, L., and T. Sargent, (2000). Recursive macroeconomic theory, The MIT Press.

Maliar, L., Maliar, S., and F. Valli, (2009). Solving the incomplete markets model with aggregate uncertainty using the Krusell-Smith algorithm, Journal of Economic Dynamics and Control 34, 42-49.

Maliar, L., S. Maliar and P. Winant, (2019). Will AI replace computational economists any time soon? CEPR working paper DP 14024.

Reiter, M., (2010). Solving the incomplete markets economy with aggregate uncertainty by backward induction, Journal of Economic Dynamics and Control 34, 28--35.

Reiter, M., (2018). HetSol: a toolkit for solving HA (and other) models. Slides.

Winberry, T., (2016). A toolbox for solving and estimating heterogeneous agent macro models. Manuscript.

Reading for Part IV, Section 5

Aguiar, M. and G. Gopinath, (2006). Defaultable debt, interest rates and the current account. Journal of International Economics 69(1), 64-83.

Arellano, C., (2008). Default risk and income fluctuations in emerging economies, American Economic Review 98(3), 690-712.

Arellano, C., Maliar, L., Maliar S. and V. Tsyrennikov, (2014). Envelope condition method with an application to default risk models, Manuscript.

Gordon, G. and S. Qui, (2015), A divide and conquer algorithm for exploiting policy function monotonicity. Manuscript, Indiana University.

Villemot, S., (2012). Accelerating the resolution of sovereign debt models using an endogenous grid method, Dynare working paper 17, CEPREMAP.

Reading for Part IV, Section 6

Cao, D. and I. Werning, (2018). Saving and dissaving with hyperbolic discounting. Econometrica 86 (3), 805-857.

Harris, C. and D. Laibson, (2001). Dynamic choices of hyperbolic consumers, Econometrica 69 (4), 935-959.

Ferris, M., K.L., Judd and K. Schmedders, (2011). Solving dynamic games with Newton's method. Manuscript.

Judd, K., 2004, Existence, uniqueness, and computational theory for time consistent equilibria: a hyperbolic discounting example. Manuscript.

Krusell, P., and A. Smith, (2003). Consumption-savings decisions with quasi-geometric discounting. Econometrica 71, 365-375.

Krusell, P., B. Kuruscu, and A. Smith, (2002). Equilibrium welfare and government policy with quasi-geometric discounting. Journal of Economic Theory 105, 42-72.

Laibson, D., A. Repetto, and J. Tobacman, (1998). Self-control and saving for retirement, Brookings Papers on Economic Activity 1, 91-172.

Maliar, L and S. Maliar, (2005). Solving the neoclassical growth model with quasi-geometric discounting: A grid-based Euler-equation method, Computational Economics 26, 163-172.

Maliar, L and S. Maliar, (2006). The neoclassical growth model with heterogeneous quasi-geometric consumers, Journal of Money, Credit, and Banking, 38(3), 635-654.

Maliar, L. and S. Maliar, (2016). Ruling out multiplicity of smooth equilibria in dynamic games: a hyperbolic discounting example. Dynamic Games and Applications 6(2), 243-261, in special issue "Dynamic Games in Macroeconomics" edited by Edward C. Prescott and Kevin L Reffett.

Readings for Part V, Section 7

Azinovic, M., G. Luca and S. Scheidegger (2019). Deep equilibrium nets. SSRN: https://ssrn.com/abstract=3393482

Duarte, V., (2018). Machine learning for continuous-time economics. SSRN: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3012602

Fernández-Villaverde, J., S. Hurtado, and G. Nuño, (2018). Financial frictions and the wealth distribution. Manuscript.

Goodfellow, I., Y. Bengio and A. Courville, (2016). Deep Learning. The MIT Press.
Hastie, T., R. Tibshirani, and J. Friedman, (2009). The Elements of Statistical Learning. Springer.
Maliar, L., S. Maliar and P. Winant, (2019). Will AI replace computational economists any time soon? CEPR working paper DP 14024.
Villa, A. and V. Valaitis (2019). Machine learning projection methods for macro-finance models, Manuscript.

Readings for Part V, Section 1

Auroba, B and J. Fernandez-Villaverde, (2014). Comparison of programming languages in economics. Manuscript.

Coleman, C., S. Lyon, L. Maliar and S. Maliar, (2017). Matlab, Python, Julia: What to choose in economics? CEPR working paper DP 13210.

Readings for Part V, Section 2

Aldrich E.M., Fernández-Villaverde, J., Ronald Gallant, A., and J. Rubio-Ramírez, (2011). Tapping the supercomputer under your desk: Solving dynamic equilibrium models with graphics processors, Journal of Economic Dynamics and Control, 35, 386-393.

Blood, P., (2011). Getting started using national computing resources. http://staff.psc.edu/blood/ICE11/XSEDE_ICE_July2011.pdf.

Cai, Y., Judd, K. L., Train, G. and S. Wright, (2013). Solving dynamic programming problems on a computational grid. NBER working paper 18714.

Creel, M., (2005). User-friendly parallel computations with econometric examples. Computational Economics 26(2), 107-128.

Creel, M., (2008). Using parallelization to solve a macroeconomic model: a parallel parameterized expectations algorithm. Computational Economics 32, 343-352.

Fernández-Villaverde, J., and D. Zarruk Valencia, (2019). A practical guide to parallelization in Economics. Manuscript.

Maliar, L., (2013). Assessing gains from parallel computation on supercomputers. Economics Bulletin 35/1, 159-167.

Lilia Maliar and Serguei Maliar, (2014). "Numerical Methods for Large Scale Dynamic Economic Models" in: Schmedders, K. and K. Judd (Eds.), *Handbook of Computational Economics*, Volume 3, Chapter 7, Amsterdam: Elsevier Science.

Nagurney, A., (1996). Parallel computation, in: H. M. Amman, D. A. Kendrick and J. Rust (Eds.), Handbook of Computational Economics, Vol. 1, Amsterdam: Elsevier, pp. 336-401.