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JEL Classification: C61, C63, C68, E31, E52

Keywords: New keynesian model, Redistribution, Tank, Interest rate rules

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This paper supersedes "Monetary Policy and Redistribution: A Look Under the Hatch with TANK".

Monetary Policy Transmission with Endogenous Central Bank Responses in TANK*

Lilia Maliar[†] Christopher Naubert[‡]

June 17, 2024

Abstract

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1 Introduction

A notable failure of the textbook representative agent New Keynesian (RANK) model is its lack of Keynesian mechanisms in the transmission of contemporaneous changes in real interest rates. As highlighted in Kaplan, Moll, and Violante (2018), monetary policy in RANK models works almost exclusively through intertemporal substitution. In the literature, this channel is often referred to as the direct effect channel. The conventional Keynesian explanation, however, emphasizes the role of changes in income in monetary policy transmission. The first round change in consumption due to intertemporal substation, the argument goes, generates a change in income which leads to additional changes in consumption.

In the paper, we study how the transmission of monetary policy innovations is affected by the endogenous response of the central bank to standard macroeconomic aggregates in a two-agent New Keynesian model (TANK) similar to that used in Bilbiie (2008) and Debortoli and Galí (2018). We focus on how the stance of monetary policy and the fraction of savers in the economy jointly affect the transmission. We consider both contemporaneous and future monetary policy innovations. In both cases, we assume that the agents learn about the innovation in the current period. One can view the exercise as studying the transmission of monetary policy "news shocks" during a period of conventional monetary policy with the central bank following a standard Taylor rule.[1]

The literature has largely focused on monetary policy innovations that generate one-for-one changes in the real interest rate (Bilbiie 2019 and Kaplan, Moll, and Violante 2018 Section I). A central bank can implement this by following a Taylor rule that fully offsets changes in expected inflation and does not respond to any other macroeconomic variables. By focusing on innovations that generate one-for-one changes in the real interest rate, one can split the response of consumption to the innovation into a direct, price effect and an indirect, income effect. With a standard Taylor rule (i.e. one that more than offsets changes in expected inflation or responds to other macroeconomic variables), a one-unit innovation in the monetary policy shock does not necessarily generate a one-unit change in the real interest rate. In our analysis, we define the direct effect of a monetary policy innovation as the part of the consumption response attributable to the innovation holding endogenous variables such as income and inflation constant. The indirect effect, therefore, captures the part of the response due to changes in income and changes in the real interest rate attributable to the central bank's endogenous response and the response of expected inflation.

When the central bank responds endogenously to macroeconomic variables, the indirect effect of a monetary policy innovation can be either positive or negative. We show that the indirect effect is negative when the response of the central bank is sufficiently strong. We provide an interpretation of this result by further decomposing the indirect effect into an indirect income effect and an indirect real rate effect. The indirect

1. Maliar and Taylor (2024) consider monetary policy "news shocks" in a RANK model.

real rate effect captures the response of consumption to changes in the real rate that are due to the central bank's response and changes in expected inflation rather than real interest rate changes that are due directly to the innovation and the exogenous shock.

Following a monetary policy innovation, an increase in income, all else equal, works to increase consumption and contributes positively to the indirect effect. Since the central bank responds to the increase in inflation, however, with sticky prices, the real interest rate also increases. The (relatively) higher real rate contributes negatively to the indirect effect. When the central bank's response is sufficiently strong, the latter effect dominates leading to a negative indirect effect.

Only agents who have the ability to save respond to a change in the real interest rate. Therefore, for a fixed monetary policy rule, the magnitude of the indirect real rate effect depends on the fraction of savers in the economy. We show that the magnitude of the indirect real rate effect declines as the share of non-savers increases. In other words, a certain parameterization of the central bank's policy rule may generate a negative indirect effect when many agents have access to savings, but the same parameterization of the rule may generate a positive indirect effect when few agents have access to savings.

The relative magnitudes of the two components of the indirect effect are also affected by the horizon of the innovation. Specifically, the magnitude of the indirect real rate effect declines relative to the indirect income effect as the horizon increases. Consequently, under a given parameterization, the indirect effect of a contemporaneous innovation may be negative while the indirect effect of an innovation in the distant future is positive.

Related Literature

Much work has been done on how heterogeneity affects monetary policy transmission in New Keynesian models (marginal propensity to consume heterogeneity in Auclert 2019, marginal propensity to bear risk heterogeneity in Kekre and Lenel 2022 and marginal propensity to invest heterogeneity in Luetticke 2021). Our work is most closely related to the literature that focuses on the role of heterogeneity in shaping the direct and indirect transmission of monetary policy. In this strand of the literature, the response of consumption to a change in the real interest rate is split into a direct, partial equilibrium effect that captures the response of consumption to changes in real interest rates holding income fixed and an indirect, general equilibrium effect that captures the response of consumption due to the induced change in income. Using this decomposition, Kaplan, Moll, and Violante (2018) show that nearly all of the transmission of a contemporaneous monetary policy innovation is due to the direct effect in a representative agent New Keynesian model. In their TANK model, the proportion of the total response of consumption attributable to the direct effect is roughly equal to the fraction of savers in the economy. Bilbiie (2020) provides a similar decomposition in an analytically tractable heterogeneous agent New Keynesian model with idiosyncratic risk (THANK). He shows that the share of the total response of consumption due to the indirect effect is amplified when income risk is countercyclical. The opposite is true when income risk is procyclical.

The present paper complements the existing literature by focusing on the direct and indirect effects of monetary policy innovations as opposed to the direct and indirect effects of real interest rate changes. When the central bank follows a standard Taylor rule, part of the change in the real interest rate following a monetary policy innovation is due to the endogenous response of the central bank. Our analysis separates the response of consumption to these changes from the response of consumption directly due to the innovation and exogenous shock.

Outline

The remainder of the paper is as follows. In Section 2, we present the linearized model. Section 3 presents our decomposition of responses to monetary policy innovations. In Section 4, we analyze the case of a contemporaneous Phillips curve. In Section 5, we consider a forward-looking Phillips curve. Section 6 concludes. The non-linear model and derivations are presented in Appendix 7.

2 Model

The model is a standard two-agent New Keynesian model with sticky prices and flexible wages as in Bilbiie (2008) and Debortoli and Galí (2018). There is a unit mass of agents. The share of non-savers or constrained agents is given by λ. These agents do not hold any assets either because they are not permitted to trade assets by assumption or because they are fully myopic. The share of savers or unconstrained agents is then given by $1-\lambda$. These agents are forward-looking and are permitted to trade in all asset markets. Both household types derive flow utility from consumption and dis-utility from supplying labor. The remaining details of the non-linear model are relegated to Appendix 7.1. In Appendix 7.2, we log-linearize the equilibrium conditions of the non-linear model. Throughout the remainder of the paper, we work with the log-linearized model.

Denote the output gap as y_t, consumption as c_t, the net inflation rate as π_t, the net nominal interest rate as i_t, the monetary policy shock as v_t and the innovation to the monetary policy shock as ε_t^v. The future sequence of innovations, $\left\{\varepsilon_{t+\tau}^v\right\}_{\tau=0}^{\infty}$, is revealed to agents in period t and is deterministic. The linearized model is characterized by an IS curve, a Taylor rule, a goods market clearing condition, a law of motion for the monetary policy shock and a Phillips curve. The first four equations are given by

$$\text{[IS Curve]} \qquad c_t = E_t\left[c_{t+1}\right] - \frac{1}{\sigma\left(1-\Phi\right)}\left(i_t - E_t\left[\pi_{t+1}\right]\right) \tag{1}$$

$$\text{[Taylor Rule]} \qquad i_t = \phi_\pi \pi_t + \phi_{E_\pi} E_t\left[\pi_{t+1}\right] + \phi_y y_t + v_t \tag{2}$$

$$\text{[Goods Market Clearing]} \qquad c_t = y_t \tag{3}$$

$$\text{[Monetary Policy Shock]} \qquad v_t = \rho_v v_{t-1} + \varepsilon_t^v \tag{4}$$

Household heterogeneity is captured by two terms in the linearized model: Φ and κ whose definitions are given by equation 119 and equation 116, respectively. The term $\frac{1}{(1-\Phi)}$ that multiplies the elasticity of intertemporal substitution, $\frac{1}{\sigma}$, in the IS curve is the elasticity of aggregate consumption to consumption of the unconstrained agent. When all agents are unconstrained, $\Phi = 0$. We limit our attention to parameterizations where $\Phi < 1$. Bilbiie (2008) refers to the case when $\Phi > 1$ as the "inverted aggregate demand" case.[2] The slope of the Phillips curve, κ, is also affected by household heterogeneity as the two types of households may make different labor supply decisions.

The particular specification of the Phillips curve depends on the details of the price adjustment frictions firms face. We consider two alternatives. Both alternatives assume that the firms discount future dividends using the stochastic discount factor of the unconstrained household. The rate of time preference of the unconstrained household is given by β. The slope of the Phillips curve is the same in both alternatives. In the first alternative, each firm faces a Rotemberg (1982) price adjustment cost where the cost depends on the price a firm sets today relative to the aggregate price level from the previous period which the firm treats as exogenous. This specification was previously used in Bilbiie (2019). Under this assumption, as shown in Appendix 7.1.4, the Phillips curve is given by equation 5a. In the second alternative, the price adjustment cost depends on the price a firm chooses today relative to the price the firm set in the previous period. As shown in Appendix 7.1.5, this setup results in the Phillips curve given in equation 5b which is the textbook New Keynesian Phillips curve (Galí 2015).

$$\begin{cases} \pi_t = \kappa y_t & \text{(5a)} \\ \pi_t = \beta E_t\left[\pi_{t+1}\right] + \kappa y_t & \text{(5b)} \end{cases}$$

We refer to the Phillips curve given by equation 5a as the "contemporaneous Phillips curve". The Phillips curve given by equation 5b is referred to as the "forward-looking Phillips curve".

3 Decomposing the Response

In this section, we start by decomposing the response of consumption to a monetary policy innovation into a direct effect and an indirect effect. We then provide a further decomposition of the indirect effect. We

2. In the inverted aggregate demand case, an increase in the real interest rate generates an increase in consumption.

use this additional decomposition to explain why the indirect effect may be negative when the central bank responds endogenously to macroeconomic aggregates as it does when following a conventional Taylor rule.

3.1 Direct and Indirect Effects

We begin by decomposing the response of consumption to a monetary policy innovation into two parts. The first part is what we call the direct effect, which is the response of consumption to an innovation in the monetary policy shock holding all endogenous variables fixed.[3] Consider an innovation to the monetary policy shock occurring $T \geq 0$ periods in the future (i.e. at time $t+T$). If the persistence of the monetary policy shock is zero, then the direct effect is computed as the partial derivative of consumption in period t with respect to the innovation to the monetary policy shock in period $t+T$, $\frac{\partial c_t}{\partial \varepsilon^v_{t+T}}$. If the shock is persistent, then the direct effect also captures the response of consumption to the shock in periods following the period in which the innovation to the shock occurs. That is, for an innovation T periods in the future, we include $\left(\frac{\partial c_t}{\partial v_{t+T+\tau}}\right)\left(\frac{dv_{t+T+\tau}}{d\varepsilon^v_{t+T}}\right)$ for $\tau \geq 0$ as part of the direct effect. Denote the steady state output gap as y^* and the steady state net inflation rate as π^*. The direct effect is given by $\left.\frac{dc_t}{d\varepsilon^v_{t+T}}\right|_{\{y_{t+\tau}=y^*\}_{\tau=0}^{\infty},\{\pi_{t+\tau}=\pi^*\}_{\tau=0}^{\infty}}$. We denote this as $\left.\frac{dc_t}{d\varepsilon^v_{t+T}}\right|_{y^*,\pi^*}$.

The second component of the response is the indirect effect. The indirect effect is the portion of the total effect, $\frac{dc_t}{d\varepsilon^v_{t+T}}$, not due to the direct effect. The indirect effect captures the response of consumption to changes in income as well as the response to changes in the real interest rate due to the central bank's endogenous response to macroeconomic variables and changes in expected inflation. Using the direct effect and total effect, we compute the direct effect share ϑ_T^{DE} and the indirect effect share ϑ_T^{IE}.

$$\vartheta_T^{DE} \triangleq \frac{\left(\left.\frac{dc_t}{d\varepsilon^v_{t+T}}\right|_{y^*,\pi^*}\right)}{\left(\frac{dc_t}{d\varepsilon^v_{t+T}}\right)} \tag{6}$$

$$\vartheta_T^{IE} \triangleq 1 - \frac{\left(\left.\frac{dc_t}{d\varepsilon^v_{t+T}}\right|_{y^*,\pi^*}\right)}{\left(\frac{dc_t}{d\varepsilon^v_{t+T}}\right)} \tag{7}$$

Consider a one-time positive innovation to the monetary policy shock occurring T periods in the future. The persistence of the monetary policy shock is given by ρ_v. The share of the aggregate consumption response in the log-linearized model apportioned to the consumption response of constrained agents is denoted by $\zeta \in (0,1)$. The direct effect of the innovation, as shown in Appendix 7.3.2, is given by

$$\left.\frac{dc_t}{d\varepsilon^v_{t+T}}\right|_{y^*,\pi^*} = -\left(\frac{\frac{1}{\sigma}\beta(1-\zeta)}{1-[(1-\Phi)(1-\zeta)\beta]\rho_v}\right) \times [(1-\Phi)(1-\zeta)\beta]^T \tag{8}$$

3. We thank an anonymous referee for suggesting this decomposition.

From equation 8, one sees that the direct effect is always negative. This follows from the fact that both Φ and ζ are positive and less than unity. Additionally, the size of the direct effect decreases with the horizon of the innovation and converges to zero in the limit.

The direct effect is a partial equilibrium effect. Therefore, the size of the direct effect is independent of the pricing frictions firms face. Additionally, the direct effect does not depend on the responsiveness of the central bank to macroeconomic aggregates.

Unlike the level of the direct effect, however, the direct effect share does depend on the other aspects of the model. In general, the direct effect share may be less than or greater than unity. A direct effect share greater than unity implies that the indirect effect is *negative*. In Section 3.2, we further decompose the indirect effect to show how the central bank's response to macroeconomic variables can generate a negative indirect effect.

3.2 Indirect Income and Indirect Real Rate Effects

Using the planned expenditure curve, one can see what drives a negative indirect effect. The planned expenditure curve expresses consumption as a function of income, y_t, and the real interest rate, $(i_t - E[\pi_{t+1}])$. Denote the forward operator as $\mathcal{F}$. For any variable x_t, we have $\mathcal{F}x_t = x_{t+1}$. As shown in Appendix 7.3, the planned expenditure curve can be written as

$$c_t = \left[\frac{1-(1-\Phi)(1-\zeta)\beta}{1-(1-\Phi)(1-\zeta)\beta\mathcal{F}}\right] y_t - \left[\frac{\frac{1}{\sigma}\beta(1-\zeta)}{1-(1-\Phi)(1-\zeta)\beta\mathcal{F}}\right](i_t - E_t[\pi_{t+1}]) \tag{9}$$

$$= \underbrace{\overbrace{\left[\frac{1-(1-\Phi)(1-\zeta)\beta}{1-(1-\Phi)(1-\zeta)\beta\mathcal{F}}\right] y_t}^{\text{Indirect Income}} - \overbrace{\left[\frac{\frac{1}{\sigma}\beta(1-\zeta)}{1-(1-\Phi)(1-\zeta)\beta\mathcal{F}}\right]([i_t - v_t] - E_t[\pi_{t+1}])}^{\text{Indirect Real Rate}}}_{\text{Indirect}} + \underbrace{\left(-\left[\frac{\frac{1}{\sigma}\beta(1-\zeta)}{1-(1-\Phi)(1-\zeta)\beta\mathcal{F}}\right] v_t\right)}_{\text{Direct}} \tag{10}$$

In equation 10, we split the indirect effect into an *indirect income* effect and an *indirect real rate* effect. The indirect income effect captures the portion of the indirect effect due to changes in income absent changes in the real interest rate. The indirect real rate effect captures the portion of the indirect effect due to changes in the real interest rate absent changes in income. The above decomposition shows that the indirect effect is negative when the magnitude of the indirect real rate effect exceeds the magnitude of the indirect income effect.

In the model with a contemporaneous Phillips curve, we can use the decomposition to analytically characterize the threshold level of the central bank's response to inflation that generates a negative indirect effect if the innovation is contemporaneous or in the distant future. We consider these cases in Section 4.1 and Section 4.2, respectively. With a forward-looking Phillips curve, the decomposition only permits an analytical characterization in the case of a contemporaneous innovation which we consider in Section 5.1. For all other cases, we rely on numerical results.

4 Contemporaneous Phillips Curve

Unlike the direct effect of a monetary policy innovation, the total effect incorporates general equilibrium feedback effects. Therefore, the total effect depends on the pricing frictions, the systematic component of monetary policy as captured by the parameters of the Taylor rule and all other model features. In this section, we consider the case when the Rotemberg (1982) adjustment cost a firm faces when resetting its price depends on the aggregate price level in the previous period which the firm takes as given as in Bilbiie (2019). In this case, the linearized Phillips curve, as shown in Appendix 7.1.4, is given by equation 5a and restated here

$$\pi = \kappa y_t \tag{11}$$

In this section, we restrict our attention to a Taylor rule that only responds to inflation

$$i_t = \phi_\pi \pi_t + v_t \tag{12}$$

As shown in Appendix 7.4, the total effect of an innovation occurring T periods in the future is

$$\frac{dc_t}{d\varepsilon^v_{t+T}} = -\left(\frac{1}{\sigma}\left[\frac{1}{\frac{1}{\sigma}\kappa\phi_\pi + (1-\Phi)}\right]\left[\frac{1}{1-\left[\frac{\frac{1}{\sigma}\kappa+(1-\Phi)}{\frac{1}{\sigma}\kappa\phi_\pi+(1-\Phi)}\rho_v\right]}\right]\right) \times \left[\frac{\frac{1}{\sigma}\kappa + (1-\Phi)}{\frac{1}{\sigma}\kappa\phi_\pi + (1-\Phi)}\right]^T \tag{13}$$

The total effect of the innovation converges to zero as $T \to \infty$ so long as $\phi_\pi > 1$. As seen in equation 13, the speed of convergence is determined by the central bank's response to inflation. Finally, the total effect is always negative. Consequently, positive monetary policy innovations are contractionary, and negative monetary policy innovations are expansionary as would be expected.

The direct effect share, ϑ^{DE}_T, is given by the ratio of the direct effect (equation 8) to the total effect (equation 13).

$$\vartheta^{DE}_T = \left(\frac{\frac{\beta(1-\zeta)}{1-[(1-\Phi)(1-\zeta)\beta]\rho_v}}{\left[\frac{1}{\frac{1}{\sigma}\kappa\phi_\pi+(1-\Phi)}\right]\left[\frac{1}{1-\left[\frac{\frac{1}{\sigma}\kappa+(1-\Phi)}{\frac{1}{\sigma}\kappa\phi_\pi+(1-\Phi)}\rho_v\right]}\right]}\right) \times \left[\frac{(1-\Phi)(1-\zeta)\beta}{\left(\frac{\frac{1}{\sigma}\kappa+(1-\Phi)}{\frac{1}{\sigma}\kappa\phi_\pi+(1-\Phi)}\right)}\right]^T \tag{14}$$

From equation 14, one sees that the direct effect share depends on the share of constrained agents, through Φ, ζ and κ; the stance of monetary policy, through ϕ_π and the horizon of the innovation, through T. The remainder of the paper analyzes how the direct effect share and, consequently, the sign of the indirect effect depend on these three dimensions.

4.1 Contemporaneous Innovation

In this section and Section 4.2, we assume that we can interchange differentiation and the application of the forward operator. In Section 4.3 we provide additional discussion of when this assumption is justified and when it leads to erroneous conclusions.

Consider a contemporaneous innovation (i.e. $T = 0$). Using market clearing, $c_t = y_t$, the Taylor rule, the Phillips curve and the fact that, in response to a contemporaneous innovation, $\frac{dc_{t+1}}{d\varepsilon_t^v} = \rho_v \frac{dc_t}{d\varepsilon_t^v}$, equation 10 can be written as

$$c_t = \underbrace{\left[\frac{1}{1-(1-\Phi)(1-\zeta)\beta\mathcal{F}}\right]\left[\overbrace{[1-(1-\Phi)(1-\zeta)\beta]}^{\text{Indirect Income}} - \overbrace{\left[\frac{1}{\sigma}\beta(1-\zeta)\right](\phi_\pi\kappa - \rho_v\kappa)}^{\text{Indirect Real Rate}}\right] y_t}_{\text{Indirect}}$$
$$\underbrace{-\left[\frac{\frac{1}{\sigma}\beta(1-\zeta)}{1-(1-\Phi)(1-\zeta)\beta\mathcal{F}}\right] v_t}_{\text{Direct}} \tag{15}$$

The indirect effect is negative when the indirect real rate effect dominates the indirect income effect

$$\overbrace{[1-(1-\Phi)(1-\zeta)\beta]}^{\text{Indirect Income}} - \overbrace{\left[\frac{1}{\sigma}\beta(1-\zeta)\right](\phi_\pi\kappa - \rho_v\kappa)}^{\text{Indirect Real Rate}} < 0 \tag{16}$$

The inequality is satisfied if the central bank's response to inflation, ϕ_π, is sufficiently strong. Define the threshold level as $\phi_\pi^{NIE}(T)$ so that $\phi_\pi > \phi_\pi^{NIE}(T)$ results in a negative indirect effect when the innovation occurs T periods in the future. The formula for $\phi_\pi^{NIE}(0)$, derived from inequality 16, is given by

$$\phi_\pi^{NIE}(0) \triangleq \frac{\sigma}{\kappa}\left[\frac{1-(1-\Phi)(1-\zeta)\beta}{\beta(1-\zeta)}\right] + \rho_v \tag{17}$$

There is a positive relationship between the share of constrained agents, λ, and $\phi_\pi^{NIE}(0)$ as shown in Figure 1. The figure partitions (λ, ϕ_π) space into regions of positive indirect effects, negative indirect effects and indeterminacy. The boundary between the positive indirect effect region and negative indirect effect region is determined by tracing out $\phi_\pi^{NIE}(0)$ as a function of λ. The orange-shaded region consists of (λ, ϕ_π) combinations where the indirect effect is negative (the direct effect share exceeds unity). The blue-shaded region is the region of the space where the indirect effect is positive (the direct effect share is less than unity). The green-shaded region contains the parameter combinations where the solution is indeterminate.

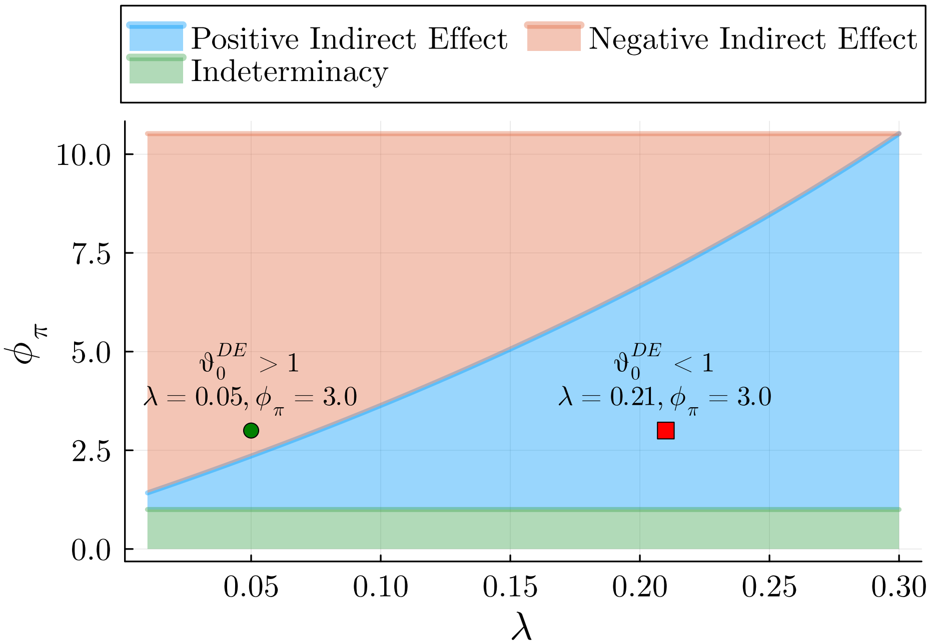

Figure 1: Contemporaneous Phillips Curve, Contemporaneous Innovation Indirect Effect Regions

Notes: The figure partitions λ (horizontal axis), ϕ_π (vertical axis) space into regions of positive indirect effects, negative indirect effects and indeterminacy. The orange-shaded region corresponds to λ and ϕ_π combinations where the indirect effect is negative. The blue-shaded region corresponds to λ and ϕ_π combinations where the indirect effect is positive. The green-shaded region corresponds to λ and ϕ_π combinations where the solution is indeterminate.

To understand why $\phi_\pi^{NIE}(0)$ increases with the share of constrained agents, λ, consider a negative monetary policy innovation. The direct effect of the innovation works to increase the consumption of the (non-myopic) unconstrained agents. To meet the higher level of desired consumption, hours worked and output both increase. These increases result in higher income and, therefore, higher consumption for both unconstrained and constrained agents. This is the indirect income effect. The central bank's endogenous response, which works to increase the real interest rate, however, has a countervailing effect, working to reduce desired consumption for the unconstrained agents. This effect is the indirect real rate effect. The question then becomes, "Which effect dominates?" With few constrained agents and many unconstrained agents (i.e. a low value of λ), many agents adjust their consumption following the central bank's endogenous response. The desire to increase consumption due to higher income is smaller in magnitude than the desire to decrease consumption due to the higher real interest rate. Therefore, the indirect real rate effect dominates, and the indirect effect is negative.

As the share of constrained agents increases (i.e. λ increases), fewer agents respond to the central bank's endogenous increase in the real interest rate. For the consumption response to the endogenous increase in the interest rate to more than offset the consumption response due to higher income, a larger increase in the interest rate is needed. In other words, the central bank needs to more aggressively respond to increases in inflation in order to achieve a negative indirect effect. Consequently, $\phi_\pi^{NIE}(0)$ increases with λ. This can be seen in Figure 1 by comparing the point $(\lambda = 0.05, \phi_\pi = 3.0)$, denoted by the green circle, with the point $(\lambda = 0.21, \phi_\pi = 3.0)$ which is denoted by the red square. In both economies, the central bank's response to inflation is the same. When only 5% of the population is constrained, the direct effect share exceeds unity which is why the green circle is in the orange, negative indirect effect region. On the other hand, when 21% of the population is constrained, the direct effect share is less than unity and the indirect effect is positive (i.e. the red square is in the blue, positive indirect effect region).

It is important to note that a negative indirect effect does not imply that the real interest rate falls following a positive monetary policy innovation.[4] Recall that the indirect real rate effect does not include the portion of the change in the real interest rate directly attributable to the monetary policy shocks. Therefore, the changes in the model's endogenous variables may generate a decline in the real rate, but, when the change in the exogenous variable is taken into account, the overall change in the real rate is positive. Mathematically, the following sequence of inequalities is possible.

$$\frac{d}{d\varepsilon_{t+T}^{v}}\left([i_t - v_t] - E_t\left[\pi_{t+1}\right]\right) < 0 < \frac{d}{d\varepsilon_{t+T}^{v}}\left(i_t - E_t\left[\pi_{t+1}\right]\right) \tag{18}$$

Figure 4 in Appendix 7.6.1 shows that the inequalities tend to hold. That is, positive contemporaneous monetary policy innovations generate real interest rate increases even when the indirect effect is negative.

Note that we can directly use $\vartheta_T^{DE}\big|_{T=0}$ to derive $\phi_\pi^{NIE}(0)$. This allows us to verify that interchanging differentiation and application of the forward operator is permissible for a given Phillips curve (contemporaneous versus forward-looking) and innovation horizon. The direct effect share, ϑ_T^{DE}, is given by the ratio of the direct effect (equation 8) to the total effect (equation 13). When the innovation is contemporaneous, the direct effect, total effect and direct effect share are given by

$$\left.\frac{dc_t}{d\varepsilon_{t+T}^{v}}\right|_{y^*,\pi^*,T=0} = -\frac{\frac{1}{\sigma}\beta\left(1-\zeta\right)}{1-\left[\left(1-\Phi\right)\left(1-\zeta\right)\beta\right]\rho_v} \tag{19}$$

$$\left.\frac{dc_t}{d\varepsilon_{t+T}^{v}}\right|_{T=0} = -\frac{1}{\sigma}\left[\frac{1}{\frac{1}{\sigma}\kappa\left(\phi_\pi-\rho_v\right)+\left(1-\Phi\right)\left(1-\rho_v\right)}\right] \tag{20}$$

$$\left.\vartheta_T^{DE}\right|_{T=0} = \frac{\beta\left(1-\zeta\right)\left[\frac{1}{\sigma}\kappa\left(\phi_\pi-\rho_v\right)+\left(1-\Phi\right)\left(1-\rho_v\right)\right]}{1-\left[\left(1-\Phi\right)\left(1-\zeta\right)\beta\right]\rho_v} \tag{21}$$

By setting equation 21 equal to unity and solving for ϕ_π, we arrive at the same bound derived previously, $\phi_\pi^{NIE}(0)$.

4. We thank an anonymous referee for bringing this point to our attention.

4.2 Innovation in the Distant Future

We now consider the case of an innovation occurring at time T with $T^* < T < \infty$ where $T^* \gg 0$.[5] When the innovation occurs $T > 1$ periods in the future, the relationship between consumption today and consumption next period is given by

$$\frac{dc_{t+1}}{d\varepsilon^v_{t+T}} = \left[\frac{\frac{1}{\sigma}\kappa\phi_\pi + (1-\Phi)}{\frac{1}{\sigma}\kappa + (1-\Phi)}\right]\frac{dc_t}{d\varepsilon^v_{t+T}} \tag{22}$$

Using this relationship, the Taylor rule, the Phillips curve and the market clearing condition in equation 10, we see that the indirect effect is negative when the following inequality is satisfied.

$$\overbrace{\left[1-(1-\Phi)(1-\zeta)\beta\right]}^{\text{Indirect Income}} \overbrace{-\left[\frac{1}{\sigma}\beta(1-\zeta)\right]\left(\phi_\pi\kappa - \left[\frac{\frac{1}{\sigma}\kappa\phi_\pi + (1-\Phi)}{\frac{1}{\sigma}\kappa + (1-\Phi)}\right]\kappa\right)}^{\text{Indirect Real Rate}} < 0 \tag{23}$$

Again, we can rearrange the inequality and solve for the threshold level for the central bank's response to inflation that leads to a negative indirect effect. Denote the threshold level as $\phi_\pi^{NIE}(\infty)$. The formula for $\phi_\pi^{NIE}(\infty)$ is given by

$$\phi_\pi^{NIE}(\infty) \triangleq \frac{\sigma}{\kappa}\left[\frac{1-(1-\Phi)(1-\zeta)\beta}{\beta(1-\zeta)}\right] + \frac{1}{(1-\Phi)(1-\zeta)\beta} \tag{24}$$

As shown in Figure 5 in Appendix 7.6, $\phi_\pi^{NIE}(\infty)$ is increasing in λ. Additionally, $\phi_\pi^{NIE}(\infty) > \phi_\pi^{NIE}(0)$ for all values of λ. Note that, unlike in the case of a contemporaneous innovation, when the innovation is in the distant future, the persistence of the shock, ρ_v, does not affect the bound. The reason why the shock persistence appears in the formula for $\phi_\pi^{NIE}(T)$ when the innovation is contemporaneous is that the persistence determines the relationship between consumption in consecutive time periods (i.e. $\frac{dc_{t+1}}{d\varepsilon^v_t} = \rho_v\frac{dc_t}{d\varepsilon^v_t}$). When the innovation occurs $T > T^*$ periods in the future, the relationship between consumption today and consumption next period is given by equation 22 which does not depend on ρ_v.

One can alternatively use the direct effect share, $\vartheta_T^{DE}\big|_{T>T^*}$, to derive the threshold. The direct effect share of an innovation in the distant future is determined by the following ratio

$$\vartheta_T^{DE}\bigg|_{T>T^*} \propto \left[\frac{(1-\Phi)(1-\zeta)\beta}{\left(\frac{\frac{1}{\sigma}\kappa+(1-\Phi)}{\frac{1}{\sigma}\kappa\phi_\pi+(1-\Phi)}\right)}\right]^T \tag{25}$$

If one sets the term in brackets equal to unity and solves for ϕ_π, one arrives at the same bound for $\phi_\pi^{NIE}(\infty)$ given in equation 24.

5. One can think of this as something similar to the limit case of taking T to infinity. In the analysis, we rely on there being a well defined relationship between consumption in consecutive periods. In much of the analysis, innovations in the infinite future have no effect on consumption today (i.e. $\frac{dc_t}{d\varepsilon^v_{t+T}}|_{T\to\infty} = 0$). Therefore, the relationship between consumption in consective periods is not well defined in this case as $c_{t+1} = ac_t$ for any $a \in \mathbb{R}$.

4.3 Innovation in the Intermediate Future

For intermediate values of T, we are unable to derive an analytical expression for the cutoff value of ϕ_π that results in a negative indirect effect, $\phi_\pi^{NIE}(T)$. In Section 4.1 and Section 4.2, we assumed that we could interchange differentiation and application of the forward operator. Performing this interchange is not problematic when considering contemporaneous innovations as the relationship between consumption in period $t+\tau$ and consumption in period $t+1+\tau$ is the same as the relationship between consumption in period t and consumption in period $t+1$

$$\frac{\left(\frac{dc_{t+1}}{d\varepsilon_t^v}\right)}{\left(\frac{dc_t}{d\varepsilon_t^v}\right)} = \frac{\left(\frac{dc_{t+1+\tau}}{d\varepsilon_t^v}\right)}{\left(\frac{dc_{t+\tau}}{d\varepsilon_t^v}\right)} = \rho_v \qquad \forall\ 0 \leq \tau < \infty$$

When the innovation is in the future, the relationship between consumption in consecutive periods is not constant. There is a constant relationship before the innovation and a constant, though different, relationship following the innovation. If the innovation is sufficiently far in the future, as was the case considered in Section 4.2, the contribution from terms dated period $t+T$ or later has a negligible effect. Therefore, we can directly use the relationship presented in equation 22 in equation 10 and ignore the forward operator.

In the intermediate future, both the contribution from future income changes to future innovations and the contribution from future income changes to past innovations have a non-negligible effect. In other words, interchanging differentiation and application of the forward operator is both mathematically incorrect and leads to incorrect conclusions. However, we can still numerically solve for the value of ϕ_π that sets the direct effect share equal to unity for $T \in \{1, ..., T^*\}$. We rely on the numerical results to verify that the positive relationship between the share of constrained agents, λ, and the cutoff level of the central bank's response to inflation which generates a negative indirect effect, $\phi_\pi^{NIE}(T)$, continues to hold when the innovation is in the intermediate future. Figure 2 plots $\phi_\pi^{NIE}(T)$ as a function of λ for $T \in \{1, 10, 50\}$. The blue line is for $T = 1$, the orange line is for $T = 10$ and the green line is for $T = 50$. For a fixed value of T, (λ, ϕ_π) combinations above the line corresponding to that value of T result in a negative indirect effect while combinations below the line result in a positive indirect effects.

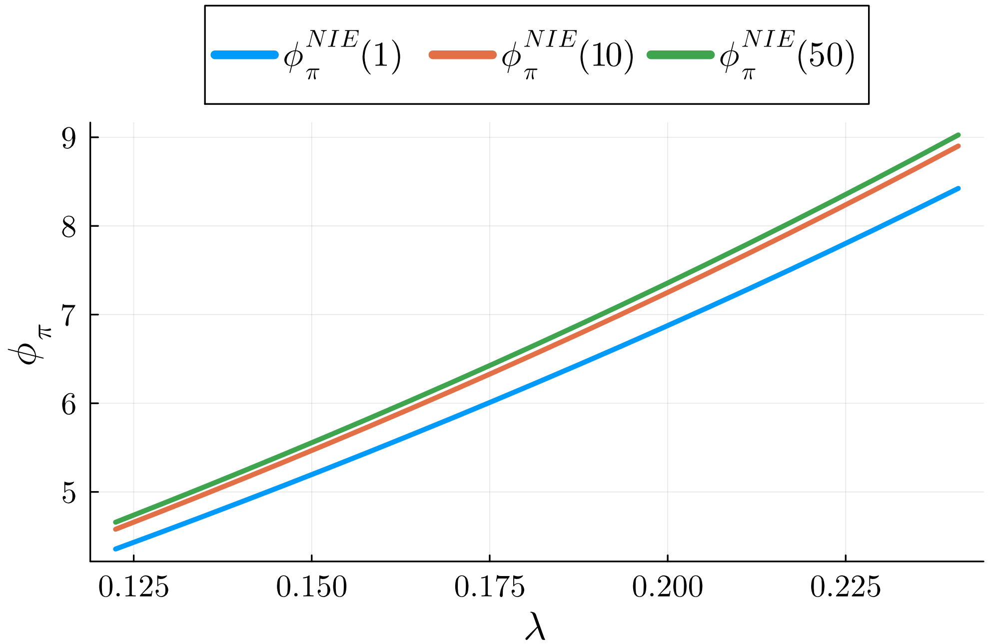

Figure 2: Threshold Inflation Response for Intermediate Values of T

Notes: The figure plots $\phi_\pi^{NIE}(T)$ as a function of λ for $T \in \{1, 10, 50\}$. The lines partition λ (horizontal axis), ϕ_π (vertical axis) space into regions of positive and negative indirect effects with each line corresponding to a different innovation horizon, T. For a fixed value of λ, ϕ_π values above a given line result in a negative indirect effect while values of ϕ_π below the line result in a positive indirect effect. The blue line is for an innovation one period in the future (i.e. $\phi_\pi^{NIE}(1)$). The orange line is for an innovation 10 periods in the future (i.e. $\phi_\pi^{NIE}(10)$). The green line is for an innovation 50 periods in the future (i.e. $\phi_\pi^{NIE}(50)$).

From the figure, one sees that the positive relationship between λ and $\phi_\pi^{NIE}(T)$ continues to hold. Additionally, we see that the threshold level for an innovation one period in the future, $\phi_\pi^{NIE}(1)$, lies uniformly below the threshold level for an innovation 10 periods in the future, $\phi_\pi^{NIE}(10)$. In other words, for a given share of constrained agents, the threshold level is increasing in T.

The reason for this result is as follows. When the innovation is in the future, changes in consumption increase in magnitude as one approaches the date of the innovation. Following the innovation, changes in consumption decrease in magnitude.

$$\frac{dc_{t+1+\tau}}{d\varepsilon^v_{t+T}} = \begin{cases} \underbrace{\left[\frac{\frac{1}{\sigma}\kappa\phi_\pi + (1-\Phi)}{\frac{1}{\sigma}\kappa + (1-\Phi)}\right]}_{>1} \frac{dc_{t+\tau}}{d\varepsilon^v_{t+T}} & \text{if } \tau < T \\ \underbrace{\rho_v}_{<1} \frac{dc_{t+\tau}}{d\varepsilon^v_{t+T}} & \text{if } \tau \geq T \end{cases} \tag{26}$$

For concreteness, consider a monetary policy innovation that increases consumption. Since future inflation is proportional to future consumption (see equation 11), all else equal, the real interest rate is lower in an arbitrary period leading up to the innovation than it is in a period following the innovation.

$$i_t - \kappa \overbrace{\left[\frac{\frac{1}{\sigma}\kappa\phi_\pi + (1-\Phi)}{\frac{1}{\sigma}\kappa + (1-\Phi)} \right] c_t}^{\pi_{t+1}} < i_t - \overbrace{\kappa\rho_v c_t}^{\pi_{t+1}} \tag{27}$$

By construction, the number of periods leading up to the innovation increases with the horizon of the innovation. Consequently, all else equal, the indirect real rate effect declines in magnitude with the horizon. Therefore, to generate an indirect real rate effect large enough in magnitude to offset the indirect income effect, the central bank needs to be more responsive to inflation as T increases. As a result, $\phi_\pi^{NIE}(T)$ is increasing in the horizon of the innovation.

5 Forward Looking Phillips Curve

We now consider the case when the Rotemberg (1982) cost a firm faces when adjusting its price depends on the price the firm set in the previous period. We present the price setting problem in Appendix 7.1.5. The linearized model generates the same Phillips curve as in the textbook New Keynesian model (Galí 2015) which we refer to as the "forward-looking Phillips curve". The linearized Phillips curve is given by equation 5b which we restate here

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa y_t \tag{28}$$

With a forward-looking Phillips curve, we can only characterize the regions of positive and negative indirect effects analytically in the case of a contemporaneous innovation, (i.e. $T = 0$). Following our analysis of a contemporaneous innovation, we provide a brief commentary on future innovations.

5.1 Contemporaneous Innovation

When the monetary policy innovation is contemporaneous, the relationship between the response of inflation in period t and the response of inflation in period $t+1$ is given by

$$\frac{d\pi_{t+1}}{d\varepsilon_t^v} = \rho_v \frac{d\pi_t}{d\varepsilon_t^v} \tag{29}$$

For the derivation, see Appendix 7.5. Using this relationship and the Phillips curve in equation 10, we see that the indirect effect is negative when the following inequality is satisfied

$$\overbrace{[1-(1-\Phi)(1-\zeta)\beta]\frac{1}{\kappa}(1-\beta\rho_v)}^{\text{Indirect Income}} - \overbrace{\left[\frac{1}{\sigma}\beta(1-\zeta)\right]\left(\phi_\pi + (\phi_{E\pi}-1)\rho_v + \phi_y \frac{1}{\kappa}(1-\beta\rho_v)\right)}^{\text{Indirect Real Rate}} < 0 \tag{30}$$

One can use the inequality to partition the parameter space into a region where a contemporaneous innovation generates a positive indirect effect and a region where a contemporaneous innovation generates a negative indirect effect. For the sake of comparison with the previous section, we only present the cutoff value of the central bank's response to inflation, ϕ_π, that generates a negative indirect effect, denoted as $\phi_\pi^{NIE,FPC}(T)$. That is, for $\phi_\pi > \phi_\pi^{NIE,FPC}(T)$ the indirect effect of an innovation T periods in the future is negative. We use the superscript "FPC" to denote that the bound corresponds to the model with the **f**orward-looking **P**hillips **c**urve.

$$\phi_\pi^{NIE,FPC}(0) \triangleq \frac{\sigma}{\kappa}\frac{[1-(1-\Phi)(1-\zeta)\beta](1-\beta\rho_v)}{\beta(1-\zeta)} - (\phi_{E\pi}-1)\rho_v - \phi_y\frac{1}{\kappa}(1-\beta\rho_v) \tag{31}$$

If the central bank only responds to inflation (i.e. $\phi_{E\pi} = \phi_y = 0$), as in Section 4, $\phi_\pi^{NIE,FPC}(0)$ can be written as

$$\phi_\pi^{NIE,FPC}(0) = \phi_\pi^{NIE}(0) - \frac{\sigma}{\kappa}\frac{[1-(1-\Phi)(1-\zeta)\beta]}{\beta(1-\zeta)}\beta\rho_v \tag{32}$$

From equation 32, we see that the threshold level that results in a negative indirect effect is lower in the model with the forward-looking Phillips curve than in the model with the contemporaneous Phillips curve so long as the monetary policy shock is persistent (i.e. $\rho_v \neq 0$). The reason for this result is that, unlike in the case of the contemporaneous Phillips curve, with the forward-looking Phillips curve, inflation today also depends on inflation next period.

In Figure 3, we again partition (λ, ϕ_π) space into three regions: a region of where a contemporaneous monetary policy innovation generates a positive indirect effect (blue region), a region where an innovation generates a negative indirect effect (orange region) and an indeterminacy region (green region).

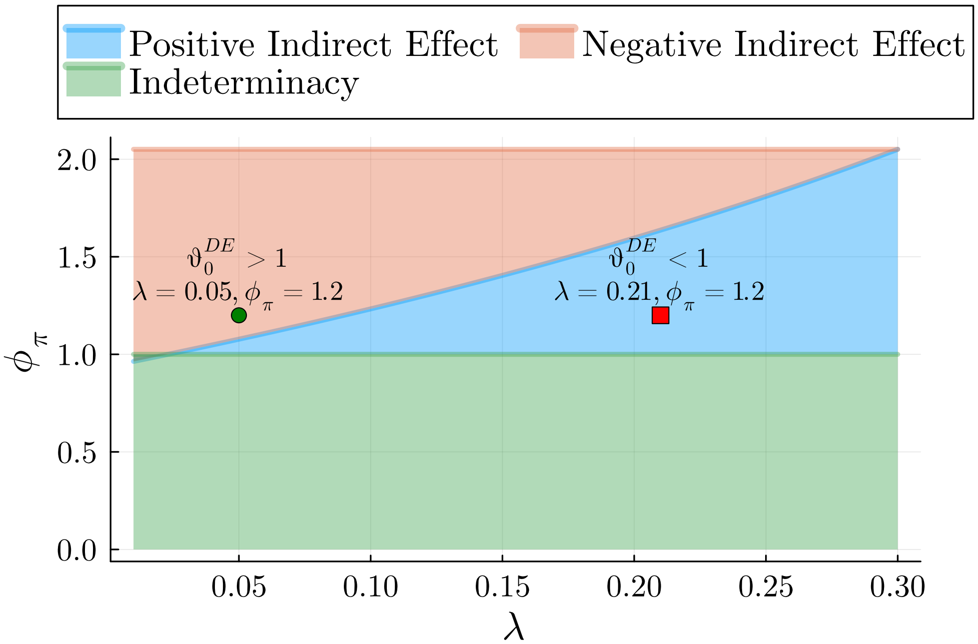

Figure 3: Forward-looking Phillips Curve, Contemporaneous Innovation Indirect Effect Regions

Notes: The figure partitions λ (horizontal axis), ϕ_π (vertical axis) space into regions of positive indirect effects, negative indirect effects and indeterminacy. The orange-shaded region corresponds to λ and ϕ_π combinations where the indirect effect is negative. The blue-shaded region corresponds to λ and ϕ_π combinations where the indirect effect is positive. The green-shaded region corresponds to λ and ϕ_π combinations where the solution is indeterminate.

The fact that the threshold in the model with the forward-looking Phillips curve, $\phi_\pi^{NIE,FPC}(0)$, is shifted downward in (λ, ϕ) space relative to the threshold in the model with the contemporaneous Phillips curve, $\phi_\pi^{NIE}(0)$, is reflected in Figure 3. As indicated by the point denoted with a green circle in Figure 3, with a forward-looking Phillips curve, a central bank that assigns a weight of 1.2 to inflation in its Taylor rule generates a negative indirect effect when 5% of the population is constrained (i.e. $\lambda = 0.05$ and $\phi_\pi = 1.2$). With a contemporaneous Phillips curve, this parameter combination resulted in a positive indirect effect as can be seen in Figure 1. The general takeaway from Figure 3 is that conventional parameter values for the central bank's response to inflation generate negative indirect effects when the model features a forward-looking Phillips curve.

5.2 A Comment on Future Innovations

With a forward-looking Phillips curve, a positive future monetary policy innovation (i.e. an innovation $T > 0$ periods in the future) no longer necessarily generates a negative total effect. In other words, positive monetary policy innovations can be expansionary, and negative monetary policy innovations can be contractionary. By responding to macroeconomic variables, the central bank undoes the effect of the initial innovation. Given the empirical implausibility of this result, we do not consider this case any further.

6 Conclusion

In this paper, we study monetary policy transmission in a standard two-agent New Keynesian model. We decompose the response of consumption to a monetary policy innovation into a direct and indirect effect. We show that the direct effect share of a monetary policy innovation may exceed unity, leading to a negative indirect effect. Our decomposition of the indirect effect shows that a negative indirect effect arises when the indirect real rate effect is larger in magnitude than the indirect income effect. The relative magnitude of the indirect real rate effect increases with the strength of the central bank's response to macroeconomic aggregates and the share of unconstrained agents. In the model with a contemporaneous Phillips curve, the magnitude of the indirect real rate effect decreases with the horizon of the innovation. In the model with a forward-looking Phillips curve, conventional values for the central bank's response to inflation and output result in a negative indirect effect following a contemporaneous monetary policy innovation.

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7 Appendix

7.1 Non-Linear Model

In this appendix, we present the non-linear model. In the model, time is discrete. The problems of the various agents are all formulated sequentially. We start with a discussion of the household block of the model. We then discuss how the government implements taxes and transfers. Next, we discuss the firm side of the model. We finish with a discussion of monetary policy and a summary of the model's equilibrium conditions. Throughout, we assume that the steady-state net inflation rate is equal to zero.

7.1.1 Households

The model is a variant of the TANK models studied in Bilbiie (2008) and Debortoli and Galí (2018). In the model, there is a continuum of agents uniformly distributed over the unit interval. Let $j \in [0,1]$ denote the name of an agent. A fraction of agents, denoted by λ, are myopic or otherwise excluded from the asset markets. We call these agents, with names $j \in [0,\lambda]$, "constrained agents" or "non-savers", and, when needed, we denote their choices with a superscript K. Those agents with names $j \in (\lambda, 1]$ are non-myopic and participate in asset markets. We call these agents "unconstrained agents" or "savers" and denote their choices with a superscript U.

Both agent types are infinitely lived. Agents have additively separable utility defined over composite consumption, $C_t^\bullet$, and hours worked, $N_t^\bullet$. Composite consumption is composed of a continuum of differentiated goods that are combined using a Dixit and Stiglitz (1977) aggregator with the elasticity of substitution given by ε.

$$C_t^\bullet = \left(\int_0^1 \left(C_t^\bullet (i) \right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{33}$$

The price of one unit of composite consumption is denoted by P_t. The utility function for consumption and dis-utility function for hours worked are both of the power form. The inverse elasticity of intertemporal substitution is given by σ. The Frisch labor supply elasticity is given by φ.

The unconstrained household discounts the future at a rate $\beta \in (0,1)$. The household receives income from supplying labor in the market. Nominal wages are denoted by W_t. Labor income is subject to a linear tax with the tax rate given by δ^W. The household also derives income from its two types of assets: one-period nominal bonds, B_{t-1}^U, and shares of a mutual fund, F_{t-1}^U. Nominal bonds pay a risk-free (gross) nominal interest rate of R_{t-1}. The mutual fund owns all the equity in the goods-producing firms. The real price of a share in the mutual fund is denoted by Q_t. Ownership of a share in the mutual fund entitles the household to a dividend payment of D_t. Dividend income is subject to a linear tax with the tax rate given by δ. The household receives transfers denoted by $T_{D,t}^U$ and $T_{W,t}^U$. The former transfer is funded by taxes on dividends while the latter transfer is funded by taxes on labor income. Note that only the unconstrained agent pays the former tax while both agents pay the latter tax. The unconstrained household uses its resources to purchase consumption, nominal bonds and mutual fund shares. The unconstrained household chooses sequences of consumption, labor, bonds and shares to maximize expected discounted utility subject to its budget constraint, initial bond holdings, initial shareholdings and initial nominal interest rate. The problem of the household is given by equations 34, 35 and 36. In equation 34, E_0 denotes the expectation conditional on the information set available at time 0.

$$\max_{\left\{C_t^U, N_t^U, B_t^U, F_t^U\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t^U\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(N_t^U\right)^{1+\varphi}}{1+\varphi} \right] \tag{34}$$

$$C_t^U + \frac{B_t^U}{P_t} + Q_t F_t^U = \frac{B_{t-1}^U R_{t-1}}{P_t} + \left(1-\delta^W\right) \frac{W_t}{P_t} N_t^U + \left[Q_t + (1-\delta) D_t\right] F_{t-1}^U + T_{D,t}^U + T_{W,t}^U \tag{35}$$

$$\left\{B_{-1}^U, F_{-1}^U, R_{-1}\right\} \text{ given} \tag{36}$$

Denote the Lagrange multiplier on the budget constraint as Λ_t^U. The unconstrained household's first-order conditions are

$$\left[C_t^U\right]: \quad \left(C_t^U\right)^{-\sigma} - \Lambda_t^U = 0 \tag{37}$$

$$\left[F_t^U\right]: \quad -\Lambda_t^U Q_t + E_t\left[\Lambda_{t+1}^U \left[Q_{t+1} + (1-\delta) D_{t+1}\right]\right] = 0 \tag{38}$$

$$\left[B_t^U\right]: \quad -\Lambda_t^U \frac{1}{P_t} + \beta R_t E_t\left[\Lambda_{t+1}^U \frac{1}{P_{t+1}}\right] = 0 \tag{39}$$

$$\left[N_t^U\right]: \quad -\left(N_t^U\right)^{\varphi} + \Lambda_t^U \left(1-\delta^W\right) \frac{W_t}{P_t} = 0 \tag{40}$$

$$\begin{aligned}\left[\Lambda_t^U\right]: \quad & \frac{B_{t-1}^U R_{t-1}}{P_t} + \left(1-\delta^W\right) \frac{W_t}{P_t} N_t^U + \left[Q_t + (1-\delta) D_t\right] F_{t-1}^U + T_{D,t}^U + T_{W,t}^U - \\ & \left[C_t^U + \frac{B_t^U}{P_t} + Q_t F_t^U\right] = 0\end{aligned} \tag{41}$$

As mentioned previously, the constrained household is myopic or lacks access to asset markets. Therefore, the constrained household solves a static problem each period. The constrained household receives labor income which is subject to the same tax rate that unconstrained households face. The only other income the constrained household receives is from transfers, $T_{D,t}^K$ and $T_{W,t}^K$. The constrained household uses its entire income to purchase consumption. The problem of the constrained household is to choose consumption and hours worked in the present period to maximize utility subject to its budget constraint. The constrained household's objective function is given in equation 42 and the budget constraint in real terms is given in equation 43.

$$\max_{\left\{C_t^K, N_t^K\right\}_{t=0}^{\infty}} \left[\frac{\left(C_t^K\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(N_t^K\right)^{1+\varphi}}{1+\varphi} \right] \tag{42}$$

$$C_t^K = \left(1-\delta^W\right) \frac{W_t}{P_t} N_t^K + T_{D,t}^K + T_{W,t}^K \tag{43}$$

Denote the Lagrange multiplier on the constrained household's budget constraint as Λ_t^K. The constrained household's first-order conditions are

$$\left[C_t^K\right]: \quad \left(C_t^K\right)^{-\sigma} - \Lambda_t^K = 0 \tag{44}$$

$$\left[N_t^K\right]: \quad -\left(N_t^K\right)^{\varphi} + \Lambda_t^K \left(1-\delta^W\right) \frac{W_t}{P_t} = 0 \tag{45}$$

$$\left[\Lambda_t^K\right]: \quad \left(1-\delta^W\right) \frac{W_t}{P_t} N_t^K + T_{D,t}^K + T_{W,t}^K - C_t^K = 0 \tag{46}$$

7.1.2 Government

The government uses taxes on dividends and labor income to fund its transfers. The amount of redistribution is determined by the tax sharing parameters, τ and τ^W. The dividend tax sharing parameter τ is restricted to the interval $[0,1]$. When $\tau = 1$, all dividend taxes are transferred to the unconstrained household. When $\tau = 0$, all agents receive an equal transfer of δD_t. We view the case of $\tau < 1$ as redistribution in favor of the constrained household as the constrained household receives more in transfers funded by dividend taxes than it contributes in dividend taxes.

For the labor income tax sharing parameter τ^W, we restrict it to the interval $\left[-\frac{1-\lambda}{\lambda}, 1\right]$. As is the case with transfers of dividend taxes, when $\tau^W = 1$, all labor income tax revenue is transferred to the unconstrained household. At the opposite end of the interval, when $\tau^W = -\frac{1-\lambda}{\lambda}$, the constrained household receives all of the labor income tax revenue. When $\tau = 0$, both households receive the same amount in transfers. Due to potential differences in labor supply, the amount received in transfers when $\tau = 0$ may differ from the amount the household pays in taxes.

The transfer rules for dividend tax revenues are given by equations 47 and 48. The transfer rules for labor income tax revenues are given by equations 49 and 50.

$$[U \text{ Dividend Transfer}] \qquad T^U_{D,t} = \left(1 + \frac{\tau\lambda}{1-\lambda}\right)\delta D_t \tag{47}$$

$$[K \text{ Dividend Transfer}] \qquad T^K_{D,t} = (1-\tau)\,\delta D_t \tag{48}$$

$$[U \text{ Labor Income Transfer}] \qquad T^U_{W,t} = \left(1 + \frac{\tau^W\lambda}{1-\lambda}\right)\delta^W \frac{W_t}{P_t} N_t \tag{49}$$

$$[K \text{ Labor Income Transfer}] \qquad T^K_{W,t} = \left(1-\tau^W\right)\delta^W \frac{W_t}{P_t} N_t \tag{50}$$

7.1.3 Firms

There is a continuum of goods-producing firms indexed by $i \in [0.1]$. The outputs produced by the different firms are imperfect substitutes. Denote firm i's output as $Y_t(i)$ and aggregate output as Y_t. The demand function for firm i's output, derived from the cost minimization problem of households in Appendix 7.1.1, is given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t \tag{51}$$

Each firm has a constant returns-to-scale production technology that uses labor to produce output. Denote labor demand of firm i as $N^d_t(i)$. Firm i's output is given by

$$Y_t(i) = N^d_t(i) \tag{52}$$

A firm hires labor at a nominal wage rate of W_t. Each firm is able to reset its price in every period subject to a Rotemberg (1982) price adjustment cost. The elasticity of the adjustment cost with respect to the net inflation rate is given by ξ. We consider two variants of the price adjustment cost which we discuss in detail in Appendix 7.1.4 and Appendix 7.1.5. Firms remit their profits to the mutual fund. Denote the real profits of firm i as $D_t(i)$ and the fraction of output going towards price adjustment costs as $\Xi_t(i) P_t$. Profits are given by

$$D_t(i) = Y_t(i)(1 - \Xi_t(i)) - \frac{W_t}{P_t} N_t^d(i) \tag{53}$$

7.1.4 Contemporaneous Phillips Curve

In this section, we derive the contemporaneous Phillips curve. Following Bilbiie (2019), we assume that the adjustment cost depends on the previous period's aggregate price level, P_{t-1}, instead of the price chosen by firm i in the previous period. In equilibrium, all firms choose the same price. Therefore, the aggregate price level is the same as the price level chosen by firm i. However, firm i treats the aggregate price level as exogenous when setting its price. Under this assumption, the price adjustment cost, denoted by $\Xi_t(i)$, is given by

$$\Xi_t(i) \triangleq \frac{\xi}{2}\left(\frac{P_t(i)}{P_{t-1}} - 1\right)^2 P_t Y_t \tag{54}$$

Firm i chooses its price to maximize the present discounted value of profits. The firm discounts using the stochastic discount factor of the unconstrained household which we denote by $\Lambda_{t,\tau}^U$. Denote firm i's real marginal cost as mc_t. The profit maximization problem of firm i is given by

$$P_t(i) =_{\hat{P}_t(i)} \sum_{\tau=t}^{\infty} \Lambda_{t,\tau}^U \left[\hat{P}_t(i) Y_t(i) - P_t mc_t Y_t(i) - \frac{\xi}{2}\left(\frac{\hat{P}_t(i)}{P_{t-1}} - 1\right)^2 P_t Y_t\right] \tag{55}$$

$$Y_t(i) = Y_t \left(\frac{\hat{P}_t(i)}{P_t}\right)^{-\varepsilon} \tag{56}$$

The first order condition with respect to $\hat{P}_t(i)$ is

$$(1-\varepsilon) Y_t(i) + \varepsilon mc_t Y_t(i) - \left[\xi \left(\frac{P_t(i)}{P_{t-1}} - 1\right) \frac{P_t}{P_{t-1}} Y_t\right] = 0 \tag{57}$$

In a symmetric equilibrium, $P_t(i) = P_t$ and $Y_t(i) = Y_t$. Denote the real markup as $\mathcal{M}_t$ and the gross inflation rate as Π_t. The Phillips curve, equation 57, can be written as

$$\begin{aligned} \Pi_t(\Pi_t - 1) &= \frac{\varepsilon}{\xi}\left(mc_t - \frac{\varepsilon - 1}{\varepsilon}\right) \\ &= \frac{\varepsilon}{\xi}\left(\frac{1}{\mathcal{M}_t} - \frac{1}{\mathcal{M}}\right) \end{aligned} \tag{58}$$

Denoting the net inflation rate as π_t and the log deviation of the markup from the steady state markup as μ_t, a first-order Taylor expansion yields

$$\pi_t = -\frac{\varepsilon}{\xi \mathcal{M}} \mu_t \tag{59}$$

7.1.5 Forward Looking Phillips Curve

In this section, we derive the forward-looking Phillips curve. We assume that the adjustment cost depends on the price chosen by firm i in the previous period. Under this assumption, the price adjustment cost, denoted by $\Xi_t(i)$, is given by

$$\Xi_t(i) \triangleq \frac{\xi}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 P_t Y_t \tag{60}$$

The profit maximization problem of firm i is given by

$$P_t(i) =_{\hat{P}_t(i)} \sum_{\tau=t}^{\infty} \Lambda_{t,\tau}^U \left[\hat{P}_t(i) Y_t(i) - P_t mc_t Y_t(i) - \frac{\xi}{2}\left(\frac{\hat{P}_t(i)}{P_{t-1}(i)} - 1\right)^2 P_t Y_t\right] \tag{61}$$

$$Y_t(i) = Y_t\left(\frac{\hat{P}_t(i)}{P_t}\right)^{-\varepsilon} \tag{62}$$

The first order condition with respect to $\hat{P}_t(i)$ is

$$\begin{aligned}(1-\varepsilon) Y_t(i) + \varepsilon mc_t Y_t(i) - \left[\xi\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)\frac{P_t}{P_{t-1}(i)} Y_t\right] + \\ \left[\Lambda_{t,t+1}^U \xi\left(\frac{P_{t+1}(i)}{P_t(i)} - 1\right)\frac{P_{t+1}}{P_t(i)} Y_{t+1}\right] = 0\end{aligned} \tag{63}$$

In a symmetric equilibrium, $P_t(i) = P_t$ and $Y_t(i) = Y_t$. Therefore, under this assumption, the Phillips curve, equation 63, can be written as

$$\begin{aligned}\Pi_t(\Pi_t - 1) &= \frac{\varepsilon}{\xi}\left(mc_t - \frac{\varepsilon - 1}{\varepsilon}\right) + E_t\left[\Lambda_{t,t+1}^U \frac{Y_{t+1}}{Y_t}\Pi_{t+1}(\Pi_{t+1} - 1)\right] \\ &= \frac{\varepsilon}{\xi}\left(\frac{1}{\mathcal{M}_t} - \frac{1}{\mathcal{M}}\right) + E_t\left[\Lambda_{t,t+1}^U \frac{Y_{t+1}}{Y_t}\Pi_{t+1}(\Pi_{t+1} - 1)\right]\end{aligned} \tag{64}$$

A first-order Taylor expansion yields

$$\pi_t = -\frac{\varepsilon}{\xi\mathcal{M}}\mu_t + \beta E_t[\pi_{t+1}] \tag{65}$$

7.1.6 Monetary Policy

To close the model, we assume that the central bank sets the nominal interest rate according to a Taylor rule. The central bank responds to deviations of inflation and expected one-period ahead inflation from target inflation and the deviation of output from its steady-state level. The strength of the central bank's responses to the deviations are given by ϕ_π, ϕ_{E_π} and ϕ_y, respectively. Denote the steady state nominal interest rate as R^* and the steady state level of output as Y^*. We assume that the target net inflation rate is zero. The monetary policy rule is subject to a shock, v_t, that follows an $AR(1)$ process. The persistence of the shock is given by ρ_v. The innovation to the shock is denoted by ε_t^v. The Taylor rule and the law of motion for the monetary policy shock are given by

$$R_t = R^* (\Pi_t)^{\phi_\pi} (E_t [\Pi_{t+1}])^{\phi_{E\pi}} \left(\frac{Y_t}{Y^*}\right)^{\phi_y} e^{v_t} \tag{66}$$

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v \tag{67}$$

The linearized Taylor rule is given by

$$i_t = \phi_\pi \pi_t + \phi_{E_\pi} E_t [\pi_{t+1}] + \phi_y y_t + v_t \tag{68}$$

7.1.7 Equilibrium Conditions

The model features 20 endogenous variables.

$$\left\{ C_t^U, F_t^U, B_t^U, N_t^U, \Lambda_t^U, T_{D,t}^U, T_{W,t}^U, C_t^K, N_t^K, \Lambda_t^K, T_{D,t}^K, T_{W,t}^K, D_t, Q_t, \Pi_t, R_t, Y_t, \mathcal{M}_t, N_t^d, \frac{W_t}{P_t} \right\}$$

The 21 equilibrium conditions, including a redundant market clearing condition, are

$$[C_t^U]: \quad (C_t^U)^{-\sigma} - \Lambda_t^U = 0 \tag{69}$$

$$[F_t^U]: \quad -\Lambda_t^U Q_t + E_t \left[\Lambda_{t+1}^U [Q_{t+1} + (1-\delta) D_{t+1}]\right] = 0 \tag{70}$$

$$[B_t^U]: \quad -\Lambda_t^U + \beta R_t E_t \left[\Lambda_{t+1}^U \frac{1}{\Pi_{t+1}}\right] = 0 \tag{71}$$

$$[N_t^U]: \quad -\left(N_t^U\right)^{\varphi} + \Lambda_t^U \left(1-\delta^W\right) \frac{W_t}{P_t} = 0 \tag{72}$$

$$[\Lambda_t^U]: \quad \frac{B_{t-1}^U R_{t-1}}{\Pi_t} + \left(1-\delta^W\right) \frac{W_t}{P_t} N_t^U + [Q_t + (1-\delta) D_t] F_{t-1}^U + T_{D,t}^U + T_{W,t}^U - \left[C_t^U + B_t^U + Q_t F_t^U\right] = 0 \tag{73}$$

$$[C_t^K]: \quad (C_t^K)^{-\sigma} - \Lambda_t^K = 0 \tag{74}$$

$$[N_t^K]: \quad -\left(N_t^K\right)^{\varphi} + \Lambda_t^K \left(1-\delta^W\right) \frac{W_t}{P_t} = 0 \tag{75}$$

$$[\Lambda_t^K]: \quad \left(1-\delta^W\right) \frac{W_t}{P_t} N_t^K + T_{D,t}^K + T_{W,t}^K - C_t^K = 0 \tag{76}$$

$$[T_{D,t}^U] \quad T_{D,t}^U - \left(1 + \frac{\tau\lambda}{1-\lambda}\right) \delta D_t = 0 \tag{77}$$

$$[T_{D,t}^K] \quad T_{D,t}^K - (1-\tau)\, \delta D_t = 0 \tag{78}$$

$$[T_{W,t}^U] \quad T_{W,t}^U - \left(1 + \frac{\tau^W \lambda}{1-\lambda}\right) \delta^W \frac{W_t}{P_t} N_t = 0 \tag{79}$$

$$[T_{W,t}^K] \quad T_{W,t}^K - \left(1-\tau^W\right) \delta^W \frac{W_t}{P_t} N_t = 0 \tag{80}$$

$$[N_t^d]: \quad \frac{1}{\mathcal{M}_t} - \frac{W_t}{P_t} = 0 \tag{81}$$

$$[\text{Phillips Curve}] \quad \left.\begin{matrix} \frac{\varepsilon}{\xi}\left(\frac{1}{\mathcal{M}_t} - \frac{1}{\mathcal{M}}\right) - \Pi_t (\Pi_t - 1) \\ \frac{\varepsilon}{\xi}\left(\frac{1}{\mathcal{M}_t} - \frac{1}{\mathcal{M}}\right) + E_t \left[\Lambda_{t,t+1}^U \frac{Y_{t+1}}{Y_t} \Pi_{t+1} (\Pi_{t+1} - 1)\right] - \Pi_t (\Pi_t - 1) \end{matrix}\right\} = 0 \tag{82}$$

$$\text{[Goods]} \qquad \lambda C_t^U + (1-\lambda) C_t^K - Y_t \left(1 - \frac{\xi}{2} (\Pi_t - 1)^2\right) = 0 \tag{83}$$

$$\text{[Labor]} \qquad \lambda N_t^K + (1-\lambda) N_t^U - N_t^d = 0 \tag{84}$$

$$\text{[Taylor Rule]} \qquad R_t - R^* (\Pi_t)^{\phi_\pi} (E_t [\Pi_{t+1}])^{\phi_{E\pi}} \left(\frac{Y_t}{Y^*}\right)^{\phi_y} e^{v_t} = 0 \tag{85}$$

$$\text{[Bond]} \qquad B_t^U = 0 \tag{86}$$

$$\text{[Shares]} \qquad F_t - 1 = 0 \tag{87}$$

$$\text{[Dividends]} \qquad Y_t \left(1 - \frac{\xi}{2} (\Pi_t - 1)^2\right) - \frac{W_t}{P_t} N_t^d - D_t = 0 \tag{88}$$

$$\text{[Production]} \qquad Y - N^d = 0 \tag{89}$$

Aggregate consumption demand, C_t, and aggregate labor supply, N_t, are given by

$$C_t \triangleq \lambda C_t^U + (1-\lambda) C_t^K \tag{90}$$

$$N_t \triangleq \lambda N_t^U + (1-\lambda) N_t^K \tag{91}$$

When log-linearizing the model in Appendix 7.2, we work with two auxiliary variables which are helpful in reducing the model to three equations. The first is the consumption gap, Γ_t.

$$\Gamma_t \triangleq 1 - \frac{C_t^K}{C_t^U} \tag{92}$$

The second is the labor gap, Ω_t.

$$\Omega_t \triangleq 1 - \frac{N_t^K}{N_t^U} \tag{93}$$

With this notation, aggregate consumption demand and aggregate labor supply can be written as

$$C_t = (1 - \lambda \Gamma_t) C_t^U \tag{94}$$

$$N_t = (1 - \lambda \Omega_t) N_t^U \tag{95}$$

In Appendix 7.7, we reduce the consumption gap to

$$\Gamma_t = \frac{\mathcal{N}_{0,t}}{\mathcal{D}_{0,t}} \tag{96}$$

where $\mathcal{N}_{0,t}$ and $\mathcal{D}_{0,t}$ are given by

$$\begin{aligned} \mathcal{N}_{0,t} =& \tau^W \delta^W + \\ & [1 - (1-\tau)\delta] [(1 - \Xi_t) \mathcal{M}_t - 1] [1 - \lambda \Omega_t] + \\ & \left[(1-\lambda)(1-\delta^w) - \lambda \tau^W \delta^W\right] \Omega_t \end{aligned} \tag{97}$$

$$\begin{aligned} \mathcal{D}_{0,t} =& \left[(1-\lambda)\left(1-\delta^W\right) + \left(1 - \lambda + \tau^W \lambda\right) \delta^W\right] + \\ & [1 - \delta(1-\tau)\lambda] [(1-\Xi_t) \mathcal{M}_t - 1] [1 - \lambda \Omega_t] - \\ & \left[1 + \left(\tau^W - 1\right)\lambda\right] \delta^W \lambda \Omega_t \end{aligned} \tag{98}$$

7.1.8 Model Parameterization

We use the following parameter values when generating the numerical results. The values are standard values used in the literature.

Table 1: Parameter Values

Parameter	Value
β	0.975
ξ	105
σ	1
φ	1
ε	10
ρ_v	0.9
τ	1
δ	0.92
τ^W	0.0
δ^W	0.15

7.2 Log-linear Equations

In this appendix, we present the log-linearized model. We start with the market clearing condition, equation 83. To a first-order approximation, Ξ_t is equal to zero. Therefore, using the definition of aggregate consumption given in equation 90, the log-linearized market clearing condition is given by

$$y_t = c_t \tag{99}$$

Using the first-order conditions for consumption and labor for the two types of households (equations 37 and 40 for the unconstrained agent and equations 44 and 45 for the constrained agent), we arrive at the following relationship between the consumption gap and the labor gap

$$(1 - \Gamma_t)^{-\sigma} = (1 - \Omega_t)^{\varphi} \tag{100}$$

Log-linearizing equation 100 gives us

$$\frac{\sigma}{1 - \Gamma}\gamma_t = -\frac{\varphi}{1 - \Omega}\omega_t \tag{101}$$

Log-linearizing equation 96 after using equation 100 to eliminate the labor gap, Ω_t, we arrive at the result that the consumption gap is proportional to the markup. The constant of proportionality, θ_1, is given in equation 186.

$$\gamma_t = \theta_1 \mu_t \tag{102}$$

The log-linearized labor first-order conditions for the two households are given by

$$w_t = \sigma c_t^U + \varphi n_t^U \tag{103}$$

$$w_t = \sigma c_t^K + \varphi n_t^K \tag{104}$$

Log-linearizing equation 95 gives us the following relationship between the labor supply of the unconstrained households, aggregate labor and the labor gap.

$$n_t^U = n_t + \frac{\lambda}{1 - \lambda\Omega}\omega_t \tag{105}$$

Similarly, we log-linearize equation 94 to drive a relationship between unconstrained consumption, aggregate consumption and the consumption gap.

$$c_t^U = c_t + \frac{\lambda}{1 - \lambda\Gamma}\gamma_t \tag{106}$$

Using equation 105 to replace n_t^U and equation 106 to replace c_t^U in the unconstrained households log-linearized first-order condition for labor (equation 103), we arrive at the following equation relating the wage to aggregate consumption, the consumption gap, aggregate labor and the labor gap.

$$w_t = \sigma c_t + \frac{\sigma\lambda}{1 - \lambda\Gamma}\gamma_t + \varphi n_t + \frac{\varphi\lambda}{1 - \lambda\Omega}\omega_t \tag{107}$$

The production function in log-linear form is

$$y_t = n_t \tag{108}$$

Log-linearizing equation 81 gives us

$$\mu_t = -w_t \tag{109}$$

Combining equations 99, 107, 108 and 109 we arrive at

$$\mu_t = -\left(\sigma + \varphi\right) y_t - \frac{\sigma\lambda}{1 - \lambda\Gamma}\gamma_t - \frac{\varphi\lambda}{1 - \lambda\Omega}\omega_t \tag{110}$$

Using the relationship between the consumption gap and labor gap given by equation 101 in equation 110 to eliminate ω_t, we have

$$\mu_t = -\left(\sigma + \varphi\right) y_t + \sigma \left[\left(\frac{\lambda}{1 - \lambda\Omega}\right)\left(\frac{1 - \Omega}{1 - \Gamma}\right) - \left(\frac{\lambda}{1 - \lambda\Gamma}\right)\right]\gamma_t \tag{111}$$

Next, we substitute for the consumption gap, γ_t, using equation 102. After rearranging, we arrive at the following relationship between the markup and output

$$\mu_t = -\left[1 - \sigma\left[\left(\frac{\lambda}{1-\lambda\Omega}\right)\left(\frac{1-\Omega}{1-\Gamma}\right) - \left(\frac{\lambda}{1-\lambda\Gamma}\right)\right]\theta_1\right]^{-1}(\sigma+\varphi)\,y_t \tag{112}$$

We denote the constant multiplying $[-(\sigma+\varphi)\,y_t]$ as θ_2

$$\theta_2 \triangleq \left[1 - \sigma\left[\left(\frac{\lambda}{1-\lambda\Omega}\right)\left(\frac{1-\Omega}{1-\Gamma}\right) - \left(\frac{\sigma\lambda}{1-\lambda\Gamma}\right)\right]\theta_1\right]^{-1} \tag{113}$$

The log-linearized Phillips curve is given by

$$\pi_t = -\frac{\varepsilon}{\xi\mathcal{M}}\mu_t \tag{114}$$

Using equation 112 and the definition of θ_2 in equation 114, we can write the Phillips curve as

$$\pi_t = \kappa y_t \tag{115}$$

where

$$\kappa \triangleq \frac{\varepsilon}{\xi\mathcal{M}}(\sigma+\varphi)\,\theta_2 \tag{116}$$

The log-linearized bond Euler equation for the unconstrained household is given by

$$c_t^U = E_t\left[c_{t+1}^U\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) \tag{117}$$

Combining equations 94, 102 and 99, we have the following relationship between consumption of the unconstrained agent and aggregate consumption

$$c_t^U = (1-\Phi)\,c_t \tag{118}$$

where Φ is defined as

$$\Phi \triangleq \frac{\lambda(\sigma+\varphi)\,\theta_1\theta_2}{1-\lambda\Gamma} \tag{119}$$

Using equation 118 in equation 117 we arrive at the following aggregate Euler equation.

$$c_t = E_t\left[c_{t+1}\right] - \frac{1}{\sigma(1-\Phi)}\left(i_t - E\left[\pi_{t+1}\right]\right) \tag{120}$$

7.3 Planned Expenditure Curve

In this appendix, we derive an expression for aggregate consumption as a function of aggregate income, the nominal interest rate and expected inflation. We use this representation to compute the direct effect of an innovation to the monetary policy shock. Additionally, this representation allows us to decompose the indirect effect into the indirect income effect and the indirect real rate effect. The derivation follows the presentation provided in the appendix of Bilbiie (2020).

7.3.1 Derivation

We start with the intertemporal budget constraint of the unconstrained household. Denote the income of the unconstrained household as Y_t^U. The household's lifetime budget constraint is given by

$$E_t\left[\sum_{i=0}^{\infty}\Lambda_{t,t+i}^{U}Y_{t+i}^{U}\right]=E_t\left[\sum_{i=0}^{\infty}\Lambda_{t,t+i}^{U}C_{t+i}^{U}\right] \tag{121}$$

Denote the log deviation of the unconstrained household's stochastic discount factor from steady state as $\hat{\Lambda}_{t,t+i}$ and the log deviation of the unconstrained household's income from steady state income as y_t^U. Log-linearizing the lifetime budget constraint gives us

$$E_t\left[\sum_{i=0}^{\infty}\beta^i\left(\hat{\Lambda}_{t,t+i}^{U}+y_{t+i}^{U}\right)\right]=E_t\left[\sum_{i=0}^{\infty}\beta^i\left(\hat{\Lambda}_{t,t+i}^{U}+c_{t+i}^{U}\right)\right] \tag{122}$$

The linearized stochastic discount factor is given by

$$\hat{\Lambda}_{t,t+i}^{U}=-\sigma\left(c_{t+i}^{U}-c_t^U\right) \tag{123}$$

Adding $\left(\frac{1}{\sigma}-1\right)E_t\left[\sum_{t=0}^{\infty}\beta^i\hat{\Lambda}_{t,t+i}\right]$ to both sides of equation 122, one obtains

$$E_t\left[\sum_{i=0}^{\infty}\beta^i\left(\frac{1}{\sigma}\hat{\Lambda}_{t,t+i}^{U}+y_{t+i}^{U}\right)\right]=E_t\left[\sum_{i=0}^{\infty}\beta^i\left(\frac{1}{\sigma}\hat{\Lambda}_{t,t+i}^{U}+c_{t+i}^{U}\right)\right] \tag{124}$$

Using the Euler equation of the unconstrained agent, the right-hand side becomes $\frac{1}{1-\beta}c_t^U$. Noting that $\hat{\Lambda}_{t,t}^U=0$, we can rewrite equation 124 as

$$E_t\left[\sum_{i=1}^{\infty}\beta^i\left(\frac{1}{\sigma}\hat{\Lambda}_{t,t+i}^{U}+y_{t+i}^{U}\right)\right]+y_t^U=\frac{1}{1-\beta}c_t^U \tag{125}$$

From the first order condition for bonds, we have

$$E_t\left[\hat{\Lambda}_{t,t+i}\right]=-E_t\left[\sum_{k=0}^{i-1}\left(i_{t+k}-\pi_{t+1+k}\right)\right] \tag{126}$$

Using this result, we have

$$\sum_{i=0}^{\infty}\beta^iE_t\left[\hat{\Lambda}_{t,t+i}\right]=-\sum_{i=1}^{\infty}\beta^iE_t\left[\sum_{k=0}^{i-1}\left(i_{t+k}-\pi_{t+1+k}\right)\right]=-\frac{\beta}{1-\beta}E_t\left[\sum_{i=0}^{\infty}\beta^i\left(i_{t+i}-\pi_{t+1+i}\right)\right] \tag{127}$$

Combining the latter two results gives us

$$\frac{1}{1-\beta}c_t^U=-\frac{1}{\sigma}\left(\frac{\beta}{1-\beta}\right)E_t\left[\sum_{i=0}^{\infty}\beta^i\left(i_{t+i}-\pi_{t+1+i}\right)\right]+E_t\left[\sum_{i=0}^{\infty}\beta^iy_{t+i}^{U}\right] \tag{128}$$

Multiplying both sides by $1-\beta$ and removing the $i=0$ variables from the summations, we have

$$\begin{aligned}c_t^U=&\left(1-\beta\right)y_t^U-\frac{1}{\sigma}\beta\left(i_t-E\left[\pi_{t+1}\right]\right)-\beta\frac{1}{\sigma}E_t\left[\sum_{i=0}^{\infty}\beta^i\left(i_{t+1+i}-\pi_{t+2+i}\right)\right]+\\&\left(1-\beta\right)E_t\left[\sum_{i=0}^{\infty}\beta^iy_{t+1+i}^{U}\right]\end{aligned} \tag{129}$$

Finally, noting that

$$\beta c_{t+1}^{U} = -\beta \frac{1}{\sigma} E_t \left[\sum_{i=0}^{\infty} \beta^i \left(i_{t+1+i} - \pi_{t+2+i} \right) \right] + (1-\beta) E_t \left[\sum_{i=0}^{\infty} \beta^i y_{t+1+i}^{U} \right] \tag{130}$$

we obtain unconstrained consumption as a function of income, the real interest rate and expected future consumption.

$$c_t^U = (1-\beta) y_t^U - \frac{1}{\sigma} \beta \left(i_t - E_t \left[\pi_{t+1} \right] \right) + \beta E_t \left[c_{t+1}^U \right] \tag{131}$$

The constrained agent consumes all of his or her income each period. Denote the log deviation of constrained income from steady-state as y_t^K. Therefore, we have the following

$$c_t^K = y_t^K \tag{132}$$

Log-linearizing equation 90 gives us the following relationship between aggregate consumption and consumption of the two types of households

$$c_t = (1-\zeta) c_t^U + \zeta c_t^K \tag{133}$$

where ζ is defined as

$$\zeta \triangleq \frac{\lambda (1-\Gamma)}{(1-\lambda) + \lambda (1-\Gamma)} \tag{134}$$

Finally, aggregating consumption across the two types of agents gives us

$$c_t = \left[1 - (1-\Phi)(1-\zeta)\beta \right] y_t - \frac{1}{\sigma} \beta (1-\zeta) \left(i_t - E_t \left[\pi_{t+1} \right] \right) + (1-\Phi)(1-\zeta) \beta E_t \left[c_{t+1} \right] \tag{135}$$

Replacing c_{t+1} with $\mathcal{F} c_t$ and rearranging gives us equation 9 in the main text.

7.3.2 Direct Effect

We now use equation 135 to derive the direct effect of an innovation occurring T periods in the future.

$$c_t = \left[1 - (1-\Phi)(1-\zeta)\beta \right] y_t - \frac{1}{\sigma} \beta (1-\zeta) \left[i_t - E_t \left[\pi_{t+1} \right] \right] + (1-\Phi)(1-\zeta) \beta E \left[c_{t+1} \right] \tag{136}$$

$$= \left[1 - (1-\Phi)(1-\zeta)\beta \right] \sum_{\tau=0}^{\infty} \left[(1-\Phi)(1-\zeta)\beta \right]^{\tau} E \left[y_{t+\tau} \right] -$$

$$\frac{1}{\sigma} \beta (1-\zeta) \sum_{\tau=0}^{\infty} \left[(1-\Phi)(1-\zeta)\beta \right]^{\tau} E \left[i_{t+\tau} - \pi_{t+1+\tau} \right] \tag{137}$$

$$= \left[1 - (1-\Phi)(1-\zeta)\beta \right] \sum_{\tau=0}^{\infty} \left[(1-\Phi)(1-\zeta)\beta \right]^{\tau} E \left[y_{t+\tau} \right] -$$

$$\frac{1}{\sigma} \beta (1-\zeta) \sum_{\tau=0}^{\infty} \left[(1-\Phi)(1-\zeta)\beta \right]^{\tau} E \left[\left(\phi_{E\pi} - 1 \right) \pi_{t+1+\tau} + \phi_y y_{t+\tau} + \phi_\pi \pi_{t+\tau} + v_{t+\tau} \right] \tag{138}$$

Assuming the persistence of the shock is ρ_v, the direct effect of an innovation happening T periods from now (in period $t+T$) is given by

$$\begin{aligned}
\left.\frac{dc_t}{d\varepsilon^v_{t+T}}\right|_{y^*,\pi^*} &= -\frac{1}{\sigma}\beta\left(1-\zeta\right)\sum_{\tau=T}^{\infty}\left[\left(1-\Phi\right)\left(1-\zeta\right)\beta\right]^{\tau}\rho_v^{\tau-T} \\
&= -\frac{1}{\sigma}\beta\left(1-\zeta\right)\left[\left(1-\Phi\right)\left(1-\zeta\right)\beta\right]^{T}\sum_{\tau=T}^{\infty}\left[\left(1-\Phi\right)\left(1-\zeta\right)\beta\right]^{\tau-T}\rho_v^{\tau-T} \\
&= -\frac{\frac{1}{\sigma}\beta\left(1-\zeta\right)\left[\left(1-\Phi\right)\left(1-\zeta\right)\beta\right]^{T}}{1-\left[\left(1-\Phi\right)\left(1-\zeta\right)\beta\right]\rho_v} \qquad (139)
\end{aligned}$$

7.4 Contemporaneous Phillips Curve: Total Effect

In this appendix, we derive the total effect of a monetary policy innovation when firms face the price setting problem presented in Appendix 7.1.4. The linearized model is characterized by the following equilibrium conditions

$$\pi_t = \kappa c_t \qquad (140)$$

$$i_t = \phi\pi_t + v_t \qquad (141)$$

$$c_t = \left[1-\left(1-\Phi\right)\left(1-\zeta\right)\beta\right]y_t - \frac{1}{\sigma}\beta\left(1-\zeta\right)\left[i_t - E_t\left[\pi_{t+1}\right]\right] + \left(1-\Phi\right)\left(1-\zeta\right)\beta E\left[c_{t+1}\right] \qquad (142)$$

$$c_t = y_t \qquad (143)$$

Denote the forward shift operator as $\mathcal{F}$. For any variable x_t, we have $\mathcal{F}x_t = x_{t+1}$. After substituting the Phillips curve, Taylor rule and market clearing condition into equation 142 and rearranging, one arrives at the following equation for consumption as a function of the sequence of exogenous variables $\{v_{t+\tau}\}_{\tau=0}^{\infty}$.

$$\begin{aligned}
c_t\left\{1-\left[1-\left(1-\Phi\right)\left(1-\zeta\right)\beta\right]+\frac{1}{\sigma}\beta\left(1-\zeta\right)\kappa\phi-\left[\frac{1}{\sigma}\beta\left(1-\zeta\right)\kappa+\left(1-\Phi\right)\left(1-\zeta\right)\beta\right]\mathcal{F}\right\} &= \\
-\frac{1}{\sigma}\beta\left(1-\zeta\right)v_t \\
c_t\left\{\left(1-\Phi\right)\left(1-\zeta\right)\beta+\frac{1}{\sigma}\beta\left(1-\zeta\right)\kappa\phi-\left[\frac{1}{\sigma}\beta\left(1-\zeta\right)\kappa+\left(1-\Phi\right)\left(1-\zeta\right)\beta\right]\mathcal{F}\right\} &= \\
-\frac{1}{\sigma}\beta\left(1-\zeta\right)v_t \\
c_t\left\{\left(1-\Phi\right)+\frac{1}{\sigma}\kappa\phi-\left[\frac{1}{\sigma}\kappa+\left(1-\Phi\right)\right]\mathcal{F}\right\} &= -\frac{1}{\sigma}v_t \\
c_t\left\{1-\left[\frac{\frac{1}{\sigma}\kappa+\left(1-\Phi\right)}{\frac{1}{\sigma}\kappa\phi+\left(1-\Phi\right)}\right]\mathcal{F}\right\} &= -\frac{1}{\sigma}\left[\frac{1}{\frac{1}{\sigma}\kappa\phi+\left(1-\Phi\right)}\right]v_t \\
c_t = -\frac{1}{\sigma}\left[\frac{1}{\frac{1}{\sigma}\kappa\phi+\left(1-\Phi\right)}\right]\sum_{\tau=0}^{\infty}\left[\frac{\frac{1}{\sigma}\kappa+\left(1-\Phi\right)}{\frac{1}{\sigma}\kappa\phi+\left(1-\Phi\right)}\right]^{\tau} v_{t+\tau}
\end{aligned}$$

Under the assumption that the monetary policy shock follows an $AR(1)$ process with persistence ρ_v, the total response of consumption to an innovation $T \geq 0$ periods in the future is

$$\frac{dc_t}{d\varepsilon_{t+T}^v} = -\frac{1}{\sigma}\left[\frac{1}{\frac{1}{\sigma}\kappa\phi + (1-\Phi)}\right]\sum_{\tau=T}^{\infty}\left[\frac{\frac{1}{\sigma}\kappa + (1-\Phi)}{\frac{1}{\sigma}\kappa\phi + (1-\Phi)}\right]^{\tau}\rho_v^{\tau-T} \tag{144}$$

$$= -\frac{1}{\sigma}\left[\frac{1}{\frac{1}{\sigma}\kappa\phi + (1-\Phi)}\right]\left[\frac{\frac{1}{\sigma}\kappa + (1-\Phi)}{\frac{1}{\sigma}\kappa\phi + (1-\Phi)}\right]^{T}\sum_{\tau=T}^{\infty}\left[\frac{\frac{1}{\sigma}\kappa + (1-\Phi)}{\frac{1}{\sigma}\kappa\phi + (1-\Phi)}\rho_v\right]^{\tau-T} \tag{145}$$

$$= -\frac{1}{\sigma}\left[\frac{1}{\frac{1}{\sigma}\kappa\phi + (1-\Phi)}\right]\left[\frac{\frac{1}{\sigma}\kappa + (1-\Phi)}{\frac{1}{\sigma}\kappa\phi + (1-\Phi)}\right]^{T}\left[\frac{1}{1-\left[\frac{\frac{1}{\sigma}\kappa + (1-\Phi)}{\frac{1}{\sigma}\kappa\phi + (1-\Phi)}\rho_v\right]}\right] \tag{146}$$

Rearranging equation 146 gives us equation 13.

7.5 Forward Looking Phillips Curve: Total Effect

We now derive the total effect in the model with the forward-looking Phillips curve. The model reduces to a linear second-order difference equation with constant coefficients. The solutions to linear second-order difference equations are well-known from the time series econometrics literature (see, for example, Sargent 1975). Denote the forward shift operator as $\mathcal{F}$. For any variable x_t, we have $\mathcal{F}x_t = x_{t+1}$. Combining the Euler equation, Phillips curve and Taylor rule, the model can be written as.

$$\left(\mathcal{F}^2 + \varpi_2\mathcal{F} + \varpi_1\right)\pi_t = -\tilde{v}_t \tag{147}$$

where

$$\varpi_1 \triangleq \left[\frac{1}{\beta} + \frac{\phi_\pi\kappa + \phi_y}{\beta\sigma(1-\Phi)}\right] \tag{148}$$

$$\varpi_2 \triangleq \left[-1 - \frac{1}{\beta} + \frac{1}{\beta\sigma(1-\Phi)}\left(\phi_{E\pi}\kappa - \beta\phi_y - \kappa\right)\right] \tag{149}$$

$$\tilde{v}_t \triangleq \frac{\kappa}{\beta\sigma(1-\Phi)}v_t \tag{150}$$

Denote the roots of the characteristic equation from equation 147 as m_1 and m_2. The formula for inflation is given by

$$\pi_t = -\frac{1}{(\mathcal{F}-m_1)(\mathcal{F}-m_2)}\tilde{v}_t \tag{151}$$

$$= \left(\frac{1}{m_1-m_2}\right)\left(\frac{1}{m_1}\right)\left(\frac{1}{1-\frac{1}{m_1}\mathcal{F}}\right)\tilde{v}_t - \left(\frac{1}{m_1-m_2}\right)\left(\frac{1}{m_2}\right)\left(\frac{1}{1-\frac{1}{m_2}\mathcal{F}}\right)\tilde{v}_t \tag{152}$$

If both roots are real and greater than unity (i.e. $m_1 > 1$ and $m_2 > 1$) or the roots are complex with magnitude greater than unity, we can rewrite this as

$$\pi_t = \left(\frac{1}{m_1-m_2}\right)\left(\frac{1}{m_1}\right)\sum_{s=0}^{\infty}(m_1)^{-s}\tilde{v}_{t+s} - \left(\frac{1}{m_1-m_2}\right)\left(\frac{1}{m_2}\right)\sum_{s=0}^{\infty}(m_2)^{-s}\tilde{v}_{t+s} \tag{153}$$

$$= \left(\frac{1}{m_1-m_2}\right)\left[\left(\frac{1}{m_1}\right)\sum_{s=t}^{\infty}m_1^{t-s}\tilde{v}_s - \left(\frac{1}{m_2}\right)\sum_{s=t}^{\infty}m_2^{t-s}\tilde{v}_s\right] \tag{154}$$

7.5.1 Real Roots

The formula for inflation is given by

$$\pi_t = \left(\frac{1}{m_1 - m_2}\right)\left[\left(\frac{1}{m_1}\right)\sum_{s=t}^{\infty} m_1^{t-s}\tilde{v}_s - \left(\frac{1}{m_2}\right)\sum_{s=t}^{\infty} m_2^{t-s}\tilde{v}_s\right] \tag{155}$$

If an innovation occurs in period $t+T$ and the shock has persistence ρ_v, the response of inflation to the innovation is

$$\frac{d\pi_t}{d\varepsilon_{t+T}^v} = \left[\frac{\kappa}{\beta\sigma(1-\Phi)}\right]\left(\frac{1}{m_1 - m_2}\right)\left[\left(\frac{1}{m_1 - \rho_v}\right)m_1^{-T} - \left(\frac{1}{m_2 - \rho_v}\right)m_2^{-T}\right] \tag{156}$$

In Section 5.1 we consider contemporaneous innovations to the monetary policy shock (i.e. $T = 0$). To compute the total effect of the innovation, we need the response of π_{t+1} to an innovation in period t. That is, we need the response of inflation to an innovation that occurred in the past. Under the maintained assumption that the shock follows an $AR(1)$ process with persistence ρ_v, the response of inflation in period $t+\tau$ to an innovation in period t is

$$\frac{d\pi_{t+\tau}}{d\varepsilon_t^v} = \left[\frac{\kappa}{\beta\sigma(1-\Phi)}\right]\left(\frac{1}{m_1 - m_2}\right)\left[\left(\frac{1}{m_1 - \rho_v}\right) - \left(\frac{1}{m_2 - \rho_v}\right)\right]\rho_v^{\tau} = \rho_v^{\tau}\frac{d\pi_t}{d\varepsilon_t^v} \tag{157}$$

7.5.2 Complex Roots

If the roots are complex, then $m_1 = m_r + im_c = re^{iw}$ and $m_2 = m_r - im_c = re^{-iw}$ where m_r is the real part and m_c is the imaginary part of m_1 and m_2. Denote the magnitude of m_1 as r and the argument as w.

$$r \triangleq \sqrt{m_r^2 + m_c^2} \tag{158}$$

$$w \triangleq \arctan\left(\frac{m_c}{m_r}\right) \tag{159}$$

The formula for inflation can be rewritten as follows

$$\pi_t = \left(\frac{1}{m_1 - m_2}\right)\left[\left(\frac{1}{m_1}\right)\sum_{s=t}^{\infty} m_1^{t-s}\tilde{v}_s - \left(\frac{1}{m_2}\right)\sum_{s=t}^{\infty} m_2^{t-s}\tilde{v}_s\right] \tag{160}$$

$$= \left(\frac{1}{m_1 - m_2}\right)\left[\sum_{s=t}^{\infty} m_1^{t-(s+1)}\tilde{v}_s - \sum_{s=t}^{\infty} m_2^{t-(s+1)}\tilde{v}_s\right] \tag{161}$$

$$= \left(\frac{1}{m_1 - m_2}\right)\left[\sum_{s=t}^{\infty} \left(m_1^{t-(s+1)} - m_2^{t-(s+1)}\right)\tilde{v}_s\right] \tag{162}$$

$$= \left(\frac{1}{m_1 - m_2}\right)\left[\sum_{s=t}^{\infty} r^{t-(s+1)}\left[\left(e^{iw}\right)^{t-(s+1)} - \left(e^{-iw}\right)^{t-(s+1)}\right]\tilde{v}_s\right] \tag{163}$$

$$= \left(\frac{1}{m_1 - m_2}\right)\left[\sum_{s=t}^{\infty} r^{t-(s+1)} 2i\sin\left[w\left(t-(s+1)\right)\right]\tilde{v}_s\right] \tag{164}$$

$$= \left(\frac{1}{r\left(e^{iw} - e^{-iw}\right)}\right)\left[\sum_{s=t}^{\infty} r^{t-(s+1)} 2i\sin\left[w\left(t-(s+1)\right)\right]\tilde{v}_s\right] \tag{165}$$

$$= \left(\frac{1}{r 2i\sin(w)}\right)\left[\sum_{s=t}^{\infty} r^{t-(s+1)} 2i\sin\left[w\left(t-(s+1)\right)\right]\tilde{v}_s\right] \tag{166}$$

$$= \left(\frac{1}{r\sin(w)}\right)\left[\sum_{s=t}^{\infty} r^{t-(s+1)}\sin\left[w\left(t-(s+1)\right)\right]\tilde{v}_s\right] \tag{167}$$

For an innovation T periods in the future, the response is

$$\frac{d\pi_t}{d\varepsilon_{t+T}^v} = \left[\frac{\kappa}{\beta\sigma(1-\Phi)}\right]\frac{1}{r\sin(w)}r^{-1-T}\sum_{s=t+T}^{\infty} r^{t+T-s}\rho_v^{s-(t+T)}\sin\left[w\left(t-1-s\right)\right] \tag{168}$$

Note that we can rewrite the sine term as follows

$$\sin\left[w\left(t-1-s\right)\right] = sin\left[w\left(T+t-s\right) + w\left(-1-T\right)\right] \tag{169}$$

$$= \sin\left[w\left(T+t-s\right)\right]\cos\left[w\left(-1-T\right)\right] + \cos\left[w\left(T+t-s\right)\right]\sin\left[w\left(-1-T\right)\right] \tag{170}$$

After substitution, we have

$$\frac{d\pi_t}{d\varepsilon_{t+T}^v} = \left[\frac{\kappa}{\beta\sigma(1-\Phi)}\right]\left[\cos\left[w\left(-1-T\right)\right]\frac{1}{r\sin(w)}r^{-1-T}\sum_{s=t+T}^{\infty}\left(\frac{\rho_v}{r}\right)^{s-(t+T)}\sin\left[w\left(T+t-s\right)\right] + \right.$$
$$\left.\sin\left[w\left(-1-T\right)\right]\frac{1}{r\sin(w)}r^{-1-T}\sum_{s=t+T}^{\infty}\left(\frac{\rho_v}{r}\right)^{s-(t+T)}\cos\left[w\left(T+t-s\right)\right]\right] \tag{171}$$

Using the properties of infinite geometric series, as shown in Appendix 7.8, we have

$$\frac{d\pi_t}{d\varepsilon_{t+T}^v} = \left[\frac{\kappa}{\beta\sigma(1-\Phi)}\right]\left[\frac{1}{r\sin(w)}r^{-1-T}\right]\left[\sin\left[w\left(-1-T\right)\right]\left[\frac{1-\frac{\rho_v}{r}\cos(w)}{1+\left(\frac{\rho_v}{r}\right)^2 - 2\left(\frac{\rho_v}{r}\right)\cos(w)}\right] - \right.$$
$$\left.\cos\left[w\left(-1-T\right)\right]\left[\frac{\frac{\rho_v}{r}\sin(w)}{1+\left(\frac{\rho_v}{r}\right)^2 - 2\left(\frac{\rho_v}{r}\right)\cos(w)}\right]\right] \tag{172}$$

7.6 Additional Figures

We include additional figures in this appendix. Appendix 7.6.1 includes additional figures for the model with the contemporaneous Phillips curve. Appendix 7.6.2 includes a figure for the model with the forward-looking Phillips curve.

7.6.1 Contemporaneous Phillips Curve

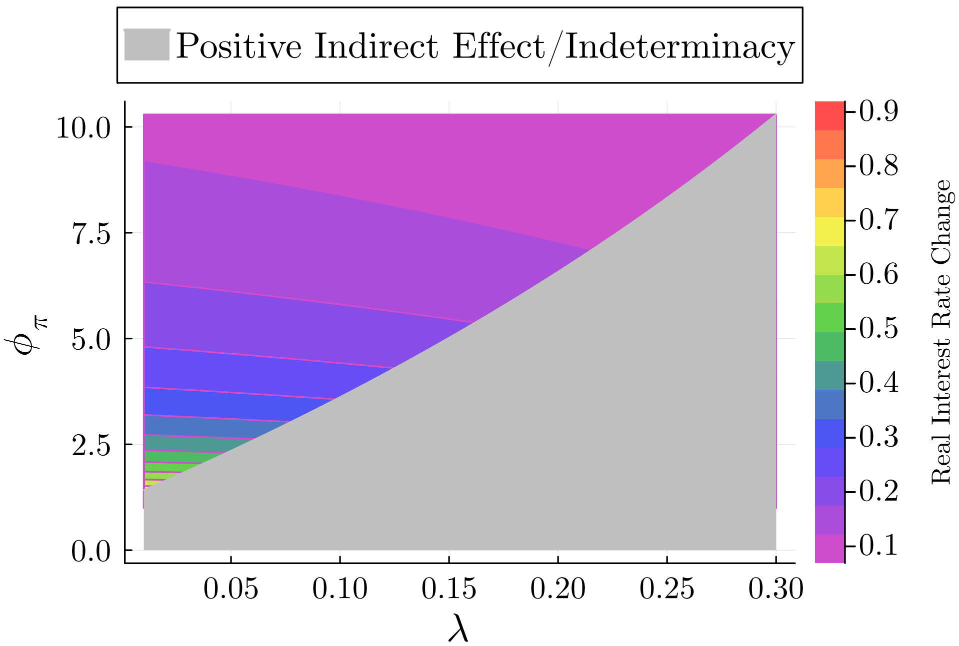

Figure 4: Real Interest Rate Change: Contemporaneous Innovation

Notes: The figure shows the pass-through of a one-unit contemporaneous innovation to the monetary policy shock. The figure partitions λ (horizontal axis), ϕ_π (vertical axis) space into a region where the solution is indeterminate or the indirect effect is positive (grey region) and a region where the indirect effect is negative (colored region). The colors of the different contours indicate the proportional pass-through of a one-unit monetary policy contemporaneous monetary policy innovation to the real interest rate. For example, a value of 0.9 means that, following a one-unit positive innovation, the real interest rate increases by 0.9 units.

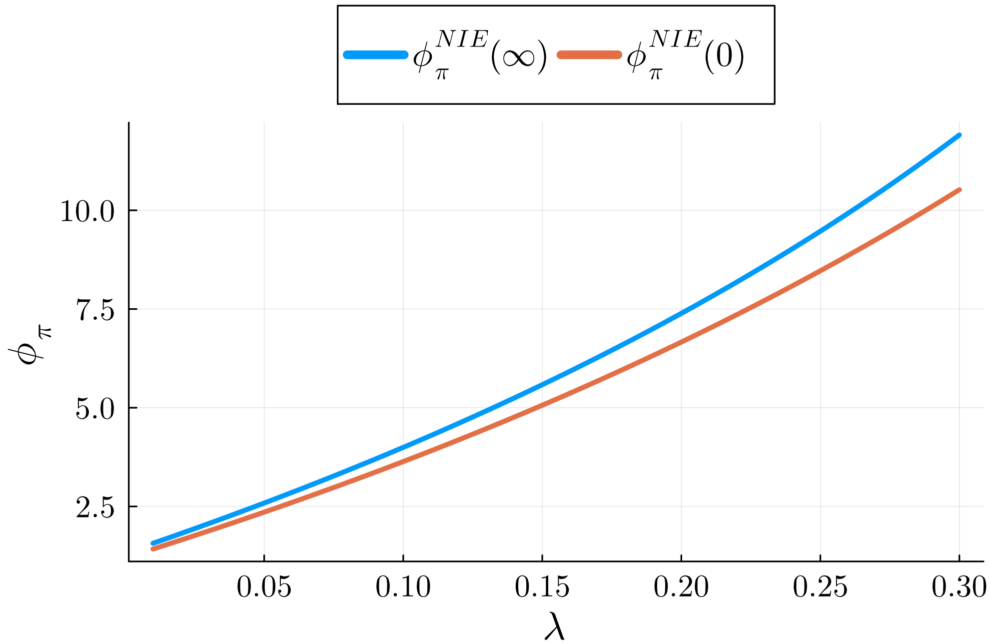

Figure 5: Indirect Effect Regions

Notes: The figure plots $\phi_\pi^{NIE}(\infty)$ and $\phi_\pi^{NIE}(0)$ as functions of λ. The lines partition λ (horizontal axis), ϕ_π (vertical axis) space into regions of positive and negative indirect effects with each line corresponding to a different innovation horizon, T. For a fixed value of λ, ϕ_π values above a given line result in a negative indirect effect while values of ϕ_π below the line result in a positive indirect effect. The blue line is for an innovation in the distant future (i.e. $\phi_\pi^{NIE}(\infty)$). The orange line is for a contemporaneous innovation (i.e. $\phi_\pi^{NIE}(0)$).

7.6.2 Forward Looking Phillips Curve

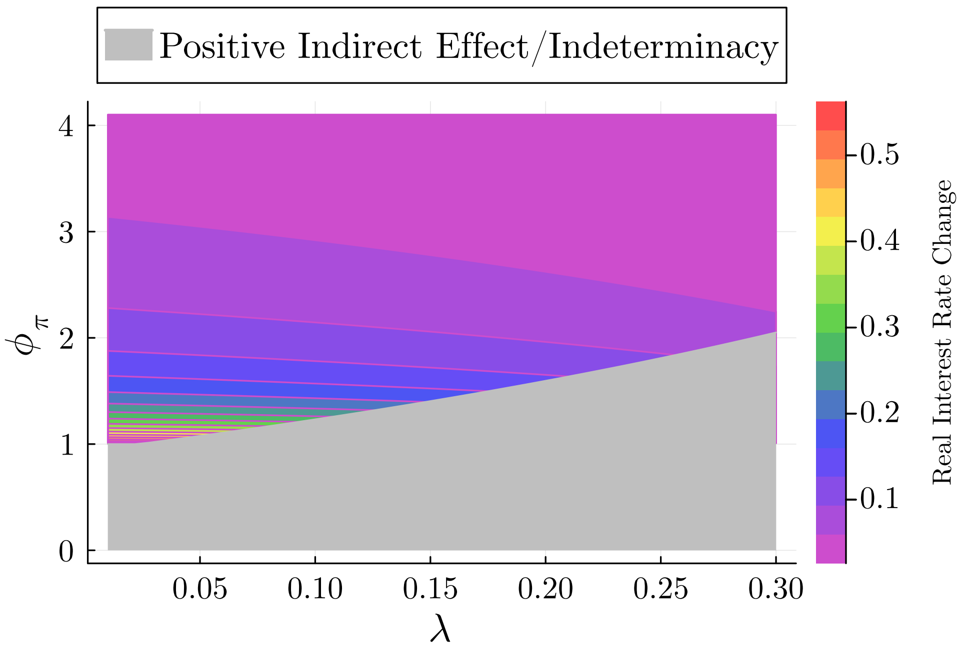

Figure 6: Real Interest Rate Change: Contemporaneous Innovation

Notes: The figure shows the pass-through of a one-unit contemporaneous innovation to the monetary policy shock. The figure partitions λ (horizontal axis), ϕ_π (vertical axis) space into a region where the solution is indeterminate or the indirect effect is positive (grey region) and a region where the indirect effect is negative (colored region). The colors of the different contours indicate the proportional pass-through of a one-unit monetary policy contemporaneous monetary policy innovation to the real interest rate. For example, a value of 0.9 means that, following a one-unit positive innovation, the real interest rate increases by 0.9 units.

7.7 Consumption Gap Derivation

Using the budget constraints of the two types of households, we can write the consumption gap, Γ_t, as

$$\Gamma_t = \frac{\left(1-\delta^W\right) W_t N_t^U + \frac{1-\delta}{1-\lambda} D_t + T_{D,t}^U + T_{W,t}^U - \left(1-\delta^W\right) W_t N_t^K - T_{D,t}^K - T_{W,t}^K}{\left(1-\delta^W\right) W_t N_t^U + \frac{1-\delta}{1-\lambda} D_t + T_{D,t}^U + T_{W,t}^U} \tag{173}$$

Next, use the definition of $T^K_{D,t}, T^K_{W,t}, T^U_{D,t}, T^U_{W,t}$ and D_t and denote the numerator as $\mathcal{N}_{0,t}$ and the denominator as $\mathcal{D}_{0,t}$. Starting with the numerator, $\mathcal{N}_{0,t}$, we have

$$\begin{aligned}
\mathcal{N}_{0,t} = &\left(1-\delta^W\right) W_t N^U_t + \frac{1-\delta}{1-\lambda} D_t + \left(1+\frac{\tau\lambda}{1-\lambda}\right)\delta D_t + \left(1+\frac{\tau^W\lambda}{1-\lambda}\right)\delta^W W_t N_t \\
&- \left(1-\delta^W\right) W_t N^K_t - (1-\tau)\,\delta D_t - \left(1-\tau^W\right)\delta^W W_t N_t \qquad (174)\\
&= \left(1-\delta^W\right) W_t N^U_t + \frac{1-\delta}{1-\lambda}\left(Y_t\left(1-\Xi_t\right) - W_t N_t\right)\\
&+ \left(1+\frac{\tau\lambda}{1-\lambda}\right)\delta\left(Y_t\left(1-\Xi_t\right) - W_t N_t\right) + \left(1+\frac{\tau^W\lambda}{1-\lambda}\right)\delta^W W_t N_t\\
&- \left(1-\delta^W\right) W_t N^K_t - (1-\tau)\,\delta\left(Y_t\left(1-\Xi_t\right) - W_t N_t\right) - \left(1-\tau^W\right)\delta^W W_t N_t \qquad (175)\\
&= \left(1-\delta^W\right) W_t N^U_t + \frac{1-\delta}{1-\lambda}\left(N_t\left(1-\Xi_t\right) - W_t N_t\right)\\
&+ \left(1+\frac{\tau\lambda}{1-\lambda}\right)\delta\left(N_t\left(1-\Xi_t\right) - W_t N_t\right) + \left(1+\frac{\tau^W\lambda}{1-\lambda}\right)\delta^W W_t N_t\\
&- \left(1-\delta^W\right) W_t N^K_t - (1-\tau)\,\delta\left(N_t\left(1-\Xi_t\right) - W_t N_t\right) - \left(1-\tau^W\right)\delta^W W_t N_t \qquad (176)\\
&= \left(1-\delta^W\right) W_t N^U_t + \frac{1-\delta}{1-\lambda}\left(\left(1-\Xi_t\right) - W_t\right) N_t\\
&+ \left(1+\frac{\tau\lambda}{1-\lambda}\right)\delta\left(\left(1-\Xi_t\right) - W_t\right) N_t + \left(1+\frac{\tau^W\lambda}{1-\lambda}\right)\delta^W W_t N_t\\
&- \left(1-\delta^W\right) W_t N^K_t - (1-\tau)\,\delta\left(\left(1-\Xi_t\right) - W_t\right) N_t - \left(1-\tau^W\right)\delta^W W_t N_t \qquad (177)
\end{aligned}$$

Divide by $W_t N^U_t$

$$\begin{aligned}
\mathcal{N}_{0,t} = &\left(1-\delta^W\right) + \frac{1-\delta}{1-\lambda}\left(\left(1-\Xi_t\right)\mathcal{M}_t - 1\right)\left(1-\lambda\Omega_t\right) + \\
&\left(1+\frac{\tau\lambda}{1-\lambda}\right)\delta\left(\left(1-\Xi_t\right)\mathcal{M}_t - 1\right)\left(1-\lambda\Omega_t\right) + \left(1+\frac{\tau^W\lambda}{1-\lambda}\right)\delta^W\left(1-\lambda\Omega_t\right)\\
&- \left(1-\delta^W\right)\left(1-\Omega_t\right) - (1-\tau)\,\delta\left(\left(1-\Xi_t\right)\mathcal{M}_t - 1\right)\left(1-\lambda\Omega_t\right) - \left(1-\tau^W\right)\delta^W\left(1-\lambda\Omega_t\right) \qquad (178)
\end{aligned}$$

Multiply through by $(1-\lambda)$

$$\begin{aligned}
\mathcal{N}_{0,t} = &(1-\lambda)\left(1-\delta^W\right) + (1-\delta)\left(\left(1-\Xi_t\right)\mathcal{M}_t - 1\right)\left(1-\lambda\Omega_t\right) + \\
&(1-\lambda+\tau\lambda)\,\delta\left(\left(1-\Xi_t\right)\mathcal{M}_t - 1\right)\left(1-\lambda\Omega_t\right) + \left(1-\lambda+\tau^W\lambda\right)\delta^W\left(1-\lambda\Omega_t\right)\\
&- (1-\lambda)\left(1-\delta^W\right)\left(1-\Omega_t\right) - (1-\lambda)(1-\tau)\,\delta\left(\left(1-\Xi_t\right)\mathcal{M}_t - 1\right)\left(1-\lambda\Omega_t\right)\\
&- (1-\lambda)\left(1-\tau^W\right)\delta^W\left(1-\lambda\Omega_t\right) \qquad (179)
\end{aligned}$$

Combining terms we have

$$\begin{aligned}
\mathcal{N}_{0,t} = &\tau^W\delta^W + \\
&\left[1-(1-\tau)\,\delta\right]\left[\left(1-\Xi_t\right)\mathcal{M}_t - 1\right]\left[1-\lambda\Omega_t\right] + \\
&\left[(1-\lambda)\left(1-\delta^w\right) - \lambda\tau^W\delta^W\right]\Omega_t \qquad (180)
\end{aligned}$$

Now turn to the denominator, $\mathcal{D}_{0,t}$

$$
\begin{aligned}
\mathcal{D}_{0,t} &= \left(1-\delta^W\right) W_t N_t^U + \frac{1-\delta}{1-\lambda} D_t + \left(1+\frac{\tau\lambda}{1-\lambda}\right)\delta D_t + \left(1+\frac{\tau^W\lambda}{1-\lambda}\right)\delta^W W_t N_t \\
&= \left(1-\delta^W\right) W_t N_t^U + \frac{1-\delta}{1-\lambda}\left(\left(1-\Xi_t\right)-W_t\right) N_t + \\
&\left(1+\frac{\tau\lambda}{1-\lambda}\right)\delta\left(\left(1-\Xi_t\right)-W_t\right) N_t + \left(1+\frac{\tau^W\lambda}{1-\lambda}\right)\delta^W W_t N_t \\
&= \left(1-\delta^W\right) W_t + \frac{1-\delta}{1-\lambda}\left(\left(1-\Xi_t\right)-W_t\right)\left[\lambda\left(1-\Omega_t\right)+\left(1-\lambda\right)\right] + \\
&\left(1+\frac{\tau\lambda}{1-\lambda}\right)\delta\left(\left(1-\Xi_t\right)-W_t\right)\left[\lambda\left(1-\Omega_t\right)+\left(1-\lambda\right)\right] + \\
&\left(1+\frac{\tau^W\lambda}{1-\lambda}\right)\delta^W W_t\left[\lambda\left(1-\Omega_t\right)+\left(1-\lambda\right)\right]
\end{aligned}
$$

Now divide through by $W_t N_t^U$

$$
\begin{aligned}
\mathcal{D}_{0,t} &= \left(1-\delta^W\right) + \frac{1-\delta}{1-\lambda}\left(\left(1-\Xi_t\right)\mathcal{M}_t-1\right)\left(1-\lambda\Omega_t\right) \\
&+ \left(1+\frac{\tau\lambda}{1-\lambda}\right)\delta\left(\left(1-\Xi_t\right)\mathcal{M}_t-1\right)\left(1-\lambda\Omega_t\right) + \left(1+\frac{\tau^W\lambda}{1-\lambda}\right)\delta^W\left(1-\lambda\Omega_t\right)
\end{aligned}
\tag{181}
$$

Next, multiply through by $(1-\lambda)$

$$
\begin{aligned}
\mathcal{D}_{0,t} =& \left(1-\lambda\right)\left(1-\delta^W\right) + \left(1-\delta\right)\left(\left(1-\Xi_t\right)\mathcal{M}_t-1\right)\left(1-\lambda\Omega_t\right) + \\
& \left(1-\lambda+\tau\lambda\right)\delta\left(\left(1-\Xi_t\right)\mathcal{M}_t-1\right)\left(1-\lambda\Omega_t\right) + \left(1-\lambda+\tau^W\lambda\right)\delta^W\left(1-\lambda\Omega_t\right)
\end{aligned}
\tag{182}
$$

Grouping together terms, we have

$$
\begin{aligned}
\mathcal{D}_{0,t} =& \left\{\left(1-\lambda\right)\left(1-\delta^W\right) + \left(1-\lambda+\tau^W\lambda\right)\delta^W\right\} + \\
& \left\{1-\delta\left(1-\tau\right)\lambda\right\}\left(\left(1-\Xi_t\right)\mathcal{M}_t-1\right)\left(1-\lambda\Omega_t\right) - \\
& \left\{1+\left(\tau^W-1\right)\lambda\right\}\delta^W\lambda\Omega_t
\end{aligned}
\tag{183}
$$

Constants/composite parameters

$$
\psi_1 \triangleq \mathcal{D}_0, \left(1-\left(1-\tau\right)\delta\right) - \mathcal{N}_0\left(1-\delta\left(1-\tau\right)\lambda\right) \tag{184}
$$

$$
\psi_2 \triangleq \mathcal{D}_0\left[\left(1-\lambda\right)\left(1-\delta^W\right)-\lambda\tau^W\delta^W\right] + \mathcal{N}_0\lambda\left(1-\left(1-\tau^W\right)\lambda\right)\delta^W \tag{185}
$$

$$
\theta_1 \triangleq \left[1+\left(\frac{1-\Omega}{1-\Gamma}\right)\left(\frac{\sigma}{\varphi}\right)\left(\frac{\psi_2-\psi_1\lambda\left(\mathcal{M}-1\right)}{\mathcal{D}_0^2}\right)\right]^{-1}\left(\frac{\psi_1\left(1-\lambda\Omega\right)}{\mathcal{D}_0^2}\right) \tag{186}
$$

$$
\theta_2 \triangleq \left[1-\sigma\left[\left(\frac{\lambda}{1-\lambda\Omega}\right)\left(\frac{1-\Omega}{1-\Gamma}\right)-\left(\frac{\sigma\lambda}{1-\lambda\Gamma}\right)\right]\theta_1\right]^{-1} \tag{187}
$$

7.8 Complex Roots Derivation

Geometric sum of sine terms

$$\sum_{k=0}^{\infty}\left(\frac{\rho_v}{r}\right)^k \sin(wk) = \frac{1}{2i}\left[\sum_{k=0}^{\infty}\left(\frac{\rho_v}{r}\right)^k e^{iwk} - \sum_{k=0}^{\infty}\left(\frac{\rho_v}{r}\right)^k e^{-iwk}\right] \tag{188}$$

$$= \frac{1}{2i}\left[\frac{1}{1-\left(\frac{\rho_v}{r}\right)e^{iw}} - \frac{1}{1-\left(\frac{\rho_v}{r}\right)e^{-iw}}\right] \tag{189}$$

$$= \frac{1}{2i}\left[\frac{1-\left(\frac{\rho_v}{r}\right)e^{-iw} - \left(1-\left(\frac{\rho_v}{r}\right)e^{iw}\right)}{\left(1-\left(\frac{\rho_v}{r}\right)e^{iw}\right)\left(1-\left(\frac{\rho_v}{r}\right)e^{-iw}\right)}\right] \tag{190}$$

$$= \frac{1}{2i}\left[\frac{\left(\frac{\rho_v}{r}\right)\left(e^{iw}-e^{-iw}\right)}{1-\left(\frac{\rho_v}{r}\right)e^{iw} - \left(\frac{\rho_v}{r}\right)e^{-iw} + \left(\frac{\rho_v}{r}\right)^2 e^{iw}e^{-iw}}\right] \tag{191}$$

$$= \left[\frac{\left(\frac{\rho_v}{r}\right)\sin(w)}{1+\left(\frac{\rho_v}{r}\right)^2 - \left(\frac{\rho_v}{r}\right)\left(e^{iw}+e^{-iw}\right)}\right] \tag{192}$$

$$= \left[\frac{\left(\frac{\rho_v}{r}\right)\sin(w)}{1+\left(\frac{\rho_v}{r}\right)^2 - 2\left(\frac{\rho_v}{r}\right)\cos(w)}\right] \tag{193}$$

Geometric sum of cosine terms

$$\sum_{k=0}^{\infty}\left(\frac{\rho_v}{r}\right)^k \cos(wk) = \frac{1}{2}\left[\sum_{k=0}^{\infty}\left(\frac{\rho_v}{r}\right)^k e^{iwk} + \sum_{k=0}^{\infty}\left(\frac{\rho_v}{r}\right)^k e^{-iwk}\right] \tag{194}$$

$$= \frac{1}{2}\left[\frac{1}{1-\left(\frac{\rho_v}{r}\right)e^{iw}} + \frac{1}{1-\left(\frac{\rho_v}{r}\right)e^{-iw}}\right] \tag{195}$$

$$= \frac{1}{2}\left[\frac{1-\left(\frac{\rho_v}{r}\right)e^{-iw} + \left(1-\left(\frac{\rho_v}{r}\right)e^{iw}\right)}{\left(1-\left(\frac{\rho_v}{r}\right)e^{iw}\right)\left(1-\left(\frac{\rho_v}{r}\right)e^{-iw}\right)}\right] \tag{196}$$

$$= \frac{1}{2}\left[\frac{2-\left(\frac{\rho_v}{r}\right)\left(e^{iw}-e^{-iw}\right)}{1-\left(\frac{\rho_v}{r}\right)e^{iw} - \left(\frac{\rho_v}{r}\right)e^{-iw} + \left(\frac{\rho_v}{r}\right)^2 e^{iw}e^{-iw}}\right] \tag{197}$$

$$= \left[\frac{1-\left(\frac{\rho_v}{r}\right)\cos(w)}{1+\left(\frac{\rho_v}{r}\right)^2 - \left(\frac{\rho_v}{r}\right)\left(e^{iw}+e^{-iw}\right)}\right] \tag{198}$$

$$= \left[\frac{1-\left(\frac{\rho_v}{r}\right)\cos(w)}{1+\left(\frac{\rho_v}{r}\right)^2 - 2\left(\frac{\rho_v}{r}\right)\cos(w)}\right] \tag{199}$$